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Hierarchical Management of Ground and Surface Water Systems Via the Multicell Approach

AN INTERIM REPORT

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HIERARCHICAL MANAGEMENT OF GROUND AND SURFACE WATER SYSTEMS VIA THE MULTICELL APPROACH

AN INTERIM REPORT FOR THE PROJECT:

Hierarchical Modeling for the Planning and Management of a Total Regional Water Resource System: Joint Consideration of the Supply and Quality of Ground and Surface Water Resources

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EXECUTIVE SUMMARY

The following are the accomplishments of the project documented in this report.

1. A modeling procedure of a complex, large-scale groundwater system via the multicell approach has been established. In Chapter 1 a sensitivity analysis of the model is discussed. It justifies its acceptance as a preferred tool for modeling aquifer systems by means of decomposition and superposition.

2. Based on the multicell concept, the parameter identification model for an aquifer system, introduced in previous phases, is further developed. The decomposition of the physical system provides for better utilization of the data base. The mathematical description of the transmissivity function is much improved when it is associated with the physical structure. A polynimial transmissivity proved to be a desirable step. Interactions between cells are considered by the application of the multicell approach.

3. Sensitivity analysis was conducted for evaluating the various approaches to the parameter identification problem. Results as are summarized in Chapter 2 of this report demonstrate the superiority of the model developed at this phase.

4. The various developments of this project provide the basis for coupling a complex, real physical system with any desired control scheme. The problem of conjunctive use of ground, surface and other water resources systems was addressed. The mathematical analysis is based on the formulation of a control problem of a distributed parameter nature. The complexity of the model is dealt with by using hierarchies of functions to relate various system responses to imposed input. Numerically the model can be solved provided those functions were made available by using a hierarchy of aquifer simulation models, resulting in a quadratic program and coordination scheme as a solution procedure.

5. The Fairfield-New Baltimore aquifer, in the lower Great Miami River Valley, served as a case study for developments in previous phases. The real data base has been further utilized for its applications to this project. The parameter identification model as well as the sensitivity analysis were tested and applied to this same area. Furthermore, multicell and particular cell simulation models for this area provided the necessary functions for applying the management model, and two results are of primary interest:

   - A prediction was obtained of water drawdowns are distributed over the aquifer for the next 10 years resulting from projected future water use.
   - An optimal plan was drawn up for well operations in Hamilton's well field located in that area.

   The scope of model testing was limited by the actual status of the area under study, but even so the potential applications of the developed model are well illustrated.

6. An example problem whereby the conjunctive use of ground and surface water systems is considered was formulated and successfully solved. The model is addressed to operations of distributed groundwater interacting with streams and operation of a related surface reservoir. The hierarchical approach proves again to be the preferred one for solving such a complex problem.

7. The field of multiobjective function analysis in water resources is of recent major interest. In a typical region like this under the surveillance of the Miami Conservancy District, Dayton, Ohio, at least three noncommensurable objectives can be defined for water use:

   - Minimize cost of water supply.
   - Minimize environmental damage caused by lowering stream water level.
   - Minimize future energy dependence of water supply.

The reported analysis and results are viewed as the first step for applying multiobjective analysis in regional water resources planning and management.

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CHAPTER 1

MODELING OF A COMPLEX, LARGE-SCALE GROUNDWATER SYSTEM THE DECOMPOSITION AND SUPERPOSITION APPROACH

1.1 INTRODUCTION

In Phase I the application of the decomposition and superposition approach as a modeling procedure for a multicell aquifer groundwater system was introduced, Haimes [1973]. A hierarchy of response functions was developed in Phase II, Haimes [1974], relating the complex system response to imposed input. The above developments laid the groundwork to practically establish mathematical models for coupling physical water systems with administrative, economical and other considerations. This study is therefore devoted to two aspects of the desired analysis:

1) To establish the multicell-particular cell simulation procedure as a major tool for large-scale groundwater system analysis. Two chapters summarize this goal. The first contains the model itself, repeated from Phase I, but with well-established procedures and mathematical justifications. The second contains the identification schemes as developed in Phases I and II but modified to use the decomposition of groundwater approach. Sensitivity analysis is applied to point out the advantages associated with the modified approach.

2) The second aspect of the desired analysis is to analyze, develop and apply a management model for the conjunctive use of a large-scale, complex groundwater system with other water resources. Three chapters

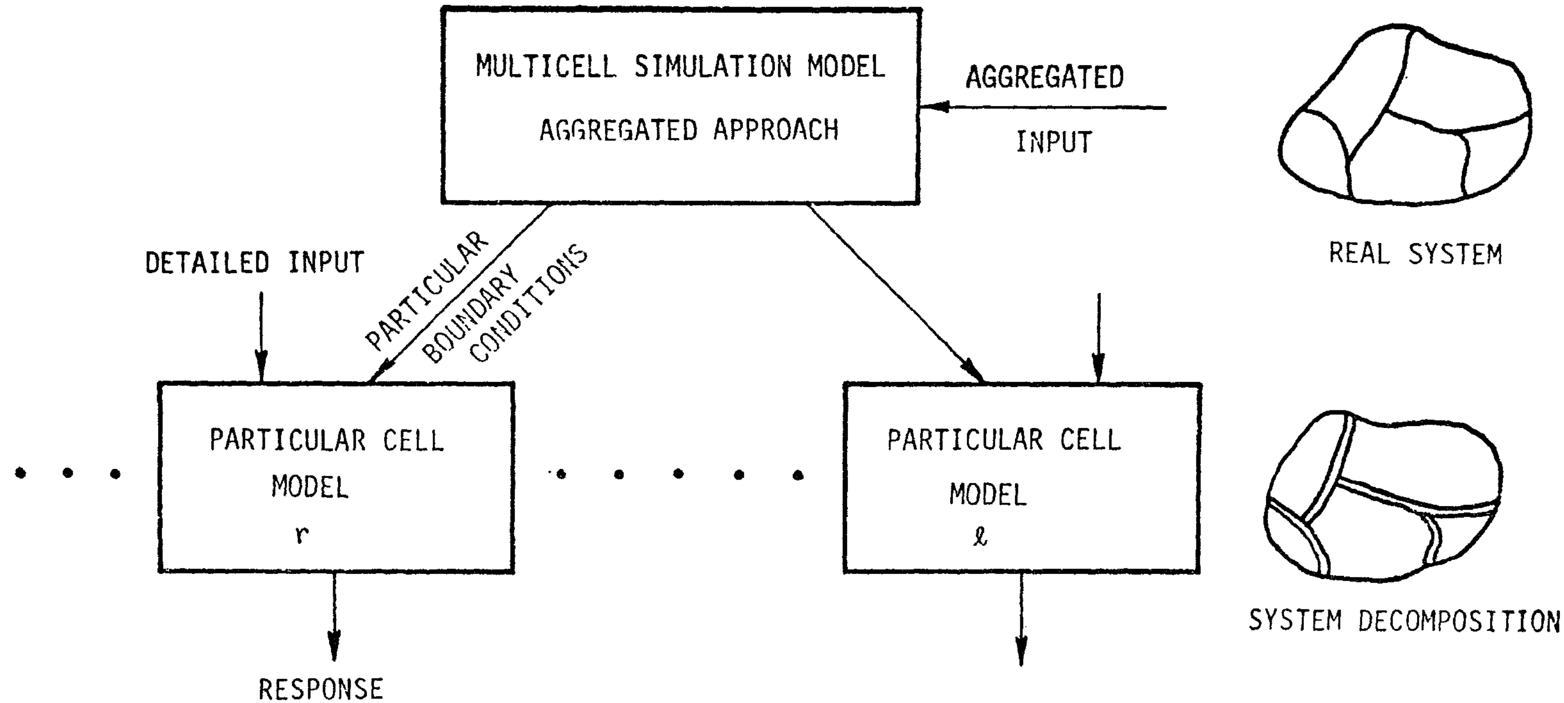
are devoted to this purpose. In one we formulate a general model where the distributed parameter system is explicitly considered. Next the model is applied to the case study described in Phases I and II, namely the Fairfield-New Baltimore area, Dayton, Ohio. Finally, based on the general model, an example problem is solved where the conjunctive use of groundwater, streams and a surface reservoir is considered. The discussion is completed by introducing a multiobjective analysis to that same area.

1.2 THE NEEDS FOR MODEL DECOMPOSITION

The groundwater simulation model plays an important role in all studies on groundwater systems: Prickett and Lonnquist, [1971], Tyson and Weber, [1964], Pinder and Bredehoeft, [1968], Bear et al, [1972], and Haimes [1973]. A simulation model will also be used in this study as the basis for developing ways of coupling the physical system with management models. However, there are many disadvantages to digital simulation models developed and used in groundwater systems modeling problems. While the traditional approach, Prickett and Lonnquist, [1971], may be appropriate for systems governed by a single partial differential equation, applying it to systems whose portions are governed effectively by different equations may make the modeling difficult. Another disadvantage occurs when the system consists of several combined unit aquifers. Although each unit is affected by the others, an input from within a unit has a greater influence than an input from outside, Haimes et al, [1968]. Thus, points within and outside a given unit deserve different weightings in the model. Finally, for any real water resources system, it is likely that detailed analysis will require extensive computer capacity followed by a considerable amount of input data which may prove to be an important restriction, Maddock, [1973]. In particular, this difficulty prevails for a large-scale aquifer system where direct use of traditional techniques may prove inadequate.

In the following a new approach to the construction of a groundwater simulation model is proposed, (Figure 1.1). The basic principle is to apply system decomposition techniques in constructing a hierarchy of simulation models. These models are aimed at determining a particular response to overall distributed activities (pumpage, recharge, etc.) throughout the system. The idea of the multicell model, Bear et al, [1972], is used farther up in the model's hierarchy for determining boundary conditions for a particular cell where the point(s) of interest has been located. The particular cell, while isolated from the rest of the system by means of the computed boundary conditions, is now modeled from an accurate analysis. This proposed modeling procedure may provide an improved solution to some of the difficulties of traditional groundwater simulation models:

1. For a large-scale, complex system, where a compact simulation model on a digital computer is evidently inadequate, the proposed technique may prove to be a real advantage. The principle of water balance equations used in formulating the multicell model provides a first approximation for the interactions between different parts of the system. Thus vertical flows as well as horizontal flows are computed along with other conditions along interfaces. These are then used as boundary conditions for decomposing the system into subsystems each of which, while isolated, is easily modeled and solved. There is no standard procedure,



Assumptions: 1) Error due to aggregation is small (function of distance).
2) Solution is unique.

FIGURE 1.1. SIMULATION MODELS HIERARCHY

however, for decomposing the system, and it is the ingenuity and experience of the system analyst that are required for an improved model structure.

2. The extensive computer capacity that is often needed introduces an important restriction to applying groundwater models. This restriction is best overcome by decomposing the model. In many cases, a groundwater simulation model is viewed as an operational tool which is used periodically. This view requires frequent running of the simulation program using mini- or middle-sized digital computers "on-line." A step-by-step procedure may permit a large-scale groundwater system to be simulated on a computer with a limited capacity.

3. The unavailability of input data with which to identify a groundwater system to be modeled by digital simulation is in most cases the main source of errors in the model's results, Bear et al, [1972]. Under a given budget for data collection, it is essentially the vicinity of the interesting area that is expected to affect the model results the most, Haimes et al, [1968]. Hence, data collection efforts should be concentrated mainly on identifying that part of the system. The proposed technique offers the advantage of considering in detail a particular cell while the rest of the system is aggregated by means of the multicell. Obviously, this advantage is greatly appreciated where the interest is on an isolated subsystem. It may not be so where interest in the system response is equally distributed over all or most of the system.

4. The hierarchy of models structure in the proposed modeling technique, (Figure 1.1), is actually not restricted by the geological or hydrological conditions of the modeled area. Hence, the lower level subsystems may be defined subject to administrative considerations. This may be desirable in cases where the groundwater model essentially couples the system with some management model where an administrative scheme controls well pumpages and artificial recharges. The advantage of having the structure of the simulation model follow that of the management model is evident.

The remainder of this chapter is devoted to a general discussion of groundwater simulation models, including the multicell model and the particular cell model comprising the model decomposition context. Some of the essential conditions and assumptions underlying the proposed technique are discussed and analyzed. Applications to a case study illustrate the procedures, pointing out the advantages of the proposed technique as opposed to other methods.

1.3 GROUNDWATER SIMULATION MODELS

A brief discussion aimed at introducing groundwater mathematical models can be found in Bear et al, [1972]. Prickett and Lonnquist, [1971] analyze digital computer aquifer simulation models more profoundly. A detailed formulation for developing groundwater simulation models is found in Pinder and Bredehoeft, [1968], regarding unsteady-state flow of a fluid in a confined aquifer. A three-dimensional flow equation system is discussed by Bredehoeft and

Pinder, [1970]. A brief list of possible mathematical models to approximate groundwater flow under different conditions is given by Haimes, [1973] , based on Bear, [1972] and others.

The common feature of most digital simulation models developed to date is that they are constructed to solve sets of equations with associated boundary conditions. These equations are assumed to describe mathematically the flow in the aquifer system. Because of the complexity of boundary conditions in the real world, no explicit solution is yet available, and hence the digital computer program is essentially for solving the mathematical model's response to a specified stress imposed on the system. The technique basically used is to solve numerically the set of equations while satisfying the boundary conditions. The procedure is to simultaneously solve the system equations, while taking into account initial and boundary conditions and the particular set of forcing functions for which the system response is desired.

The discussion in Section 1.2 on the disadvantages of commonly used simulation models relates directly to the above approach.

The decomposition approach however, suggests a different way of solving the same mathematical model, arriving at the solution in a step-by-step procedure. In that procedure, the final step corresponds to the solution of the so-called particular cell model. The solution to this model is possibly subject to boundary conditions determined by previous steps via the multicell model. The mathematical model which is used in our study is now represented.

Darcy's law and Jacob, [1950], provided Pinder and Bredehoeft, [1968] the basis for showing that for two-dimensional laminar flow in an anisotropic, non-homogeneous porous medium, the hydraulic head $h(x,y,t)$ is given by the partial differential equation

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} + q(x,y,t) \tag{1.1}$$

where $T(x,y)$ is the transmissivity coefficient, $S(x,y)$ is the storage coefficient, and $q(x,y,t)$ is the flow per unit of aquifer depth leaving the aquifer. For a particular cell model the term $q(x,y,t)$ represents the net effect of recharge and discharge from the aquifer cell. In the following discussion we assume that induced in this term are pumpages from wells and flows into and out of the cell due to interactions with its neighbors.

define

$$q(x,y,t) = \sum_{k=1}^{M} Q(k,t)\,\delta(x-x_k)\,\delta(y-y_k) + \sum_{j=1}^{J} W(j,t)\,\delta(x-x_j)\,\delta(y-y_j) \tag{1.2}$$

where $Q(k,t)$ is the pumpage at well k and $W(j,t)$ is the flow leaving the cell through the j^{th} section of the boundary line defined between the cell and its neighbors, at time t. δ is the Dirac delta function. $W(j,t)$ is determined by the multicell model, and its derivation is shown in the following section, for all boundary line sections j, $j = 1, \ldots, J$.

In addition to the boundary lines j, $j = 1,\ldots,J$, the aquifer cell may contain no-flow boundaries which we denote by λ, so that

$$\frac{\partial h(\lambda,t)}{\partial n} = 0 \tag{1.3}$$

where n is the normal direction to the boundary and $\frac{\partial h}{\partial n}$ is evaluated on the boundary.

We also denote by μ the boundary line where constant head boundaries are induced on the aquifer cell so that

$$h(\mu,t) = h(\mu) \quad , \qquad t \in [0,T] \tag{1.4}$$

the initial conditions are

$$h(x,y,0) = g(x,y) \tag{1.5}$$

corresponding to conditions before any external activity is imposed on the system.

The finite difference approach is discussed by Pinder and Bredehoeft, [1968], and others, using the alternating direction implicit iterative procedure (Peaceman and Rachford, [1955]), for solving the model equations. In our study, the simulation program developed by Maddock, [1969], was used for the case study verification, and applied to the particular cell for its solution.

We are now in a position to assume that the set of equations defined by (1.1) - (1.5) is specified and that the only information necessary to completely solve the model is the flow function $W(j,t)$, for some j and all $t \in [0,T]$. We next show that the multicell model may assist in deriving this function.

1.4 MULTICELL MODEL FORMULATION

The multicell approach to modeling groundwater makes use of a set of water balance equations, of which each represents a mass balance applied to a particular cell. For a single cell representing an area within an aquifer and surrounded by impervious boundaries, the balance equation takes the form, Bear et al, [1972]:

$$Q \cdot \Delta t = [h(t + \Delta t) - h(t)] \cdot A \cdot S$$

where

Δt = period for which the balance is written

Q = net inflow into the cell

A = area of cell

$h(t)$ = average water level elevation in the cell at time t

S = aquifer storativity at the cell (averaged)

Applying the same principle of water balance to a multicell system, taking into account the interflow between adjacent cells, leads to a set of difference equations [Bear and others]. The form of these equations is identical to the form of those which result from the discretization of a partial differential equation used to approximate the aquifer system.

The thickness of an aquifer usually is small compared with its lateral dimensions. For an unconfined flow in non-homogeneous medium, in which the storage coefficient is assumed to be independent of water table elevation while transmissivity is not, the following difference equation for the r^{th} cell and the $m+1$ period may be used, Yu and Haimes, [1974]:

$$\sum_{\ell} \{R_{\ell,r}[h(\ell,i) - h(r,i)] + U_{\ell,r}[(h(\ell,i))^2 - (h(r,i))^2]\}$$

$$= V_r[h(r,i+1) - h(r,i)] + Q(r,i) \qquad (1.7)$$

where

$$R_{\ell,r} \triangleq \frac{W_{\ell,r}\, C_{\ell,r}}{L_{\ell,r}} \qquad U_{\ell,r} \triangleq \frac{W_{\ell,r}\, K_{\ell,r}}{2 \cdot L_{\ell,r}}$$

$$V_r \triangleq \frac{A_r\, S_r}{\Delta t} \qquad C_{\ell,r} \triangleq K_{\ell,r}(E_{\ell,r} - F_{\ell,r})$$

$h(\ell,i)$ = water table elevation at the ℓ^{th} cell during the i^{th} time step

$Q(r,i)$ = net outflow from the r^{th} cell during the i^{th} time step

$W_{\ell,r}$ = length of the perpendicular sector associated with the segment between cells ℓ and r .

$L_{\ell,r}$ = distance between the centers of nodes ℓ and r.

$K_{\ell,r}$ = hydraulic conductivity averaged between cells ℓ and r.

$E_{\ell,r}$ = effective aquifer depth averaged between cells ℓ and r .

$F_{\ell,r}$ = elevation at the top of the aquifer averaged between cells ℓ and r

A_r = area of r^{th} cell

S_r = storage coefficient averaged over the r^{th} cell

The non-linear term on the left in Equation (1.7) stands for the flow from the neighboring ℓ^{th} cell into the r^{th} cell during the i^{th} period.

The first term on the right side is the quantity of water stored in the r^{th} cell during one period while the second term is the pumping flow rate from the r^{th} cell during the i^{th} period. Hence, equation (1.7) states a balance condition for the sum of all flows entering a cell from its surroundings as balanced by storage and pumpage.

One should note that the multicell approach is an over-simplification of the real system. Boundary conditions must be simplified as well. Constant flow may be handled through inflow to a particular cell. Constant head requires a fixed head for the cell at all times. No-flow requires that the hydraulic conductivity be set at zero between cells and the construction of an imaginary neighboring cell.

The multicell model provides approximate inflows and outflows for each cell in the modeling procedure. These values may be computed for each time step together with averaged water heads.

The flow between the ℓ^{th} cell and the r^{th} cell during time period i is:

$$R_{\ell,r}[h(\ell,i) - h(r,i)] + U_{\ell,r}[(h(\ell,i))^2 - (h(r,i))^2] \qquad (1.8)$$

Equation (1.8) is essentially the required flow function $W(j,t)$ (Equation (1.2)) where j corresponds to a particular neighboring cell, ℓ.

For the particular cell, a more detailed formulation may be used, and the above computed flow is then distributed along the boundary line according to spatial and hydrological considerations.

In the following section we shall state and prove the mathematical ground for the proposed procedure.

1.5 ANALYTICAL JUSTIFICATION FOR MODEL SUPERPOSITION

A new groundwater simulation procedure was developed and stated in the previous sections. System decomposition and response superposition are featured in that approach, together with input aggregation and crude approximations of some of the functions such as W(j,t) (Equation (1.2)). In the following we state and prove some of the arguments essentially underlying the basics of the proposed technique.

1.5.1 An Error Function and the Aggregation via the Multicell Model

The time-dependent effect of activities such as pumping or recharge imposed on an aquifer is distributed unequally throughout the system. In particular, at time $t > 0$, the response distribution depends on the aquifer physical characteristics, namely transmissivity and storativity (T,S) coefficients, the boundary conditions and the distance between the activated point and the interesting point, Bear, [1972]. In developing the modeling superposition procedure, a basic assumption is that the response is strongly influenced by near-well properties rather than by those further away, Haimes et al [1968]. Consequently the groundwater simulation model structure provides aggregation of pumpages in all other cells. Pumping from wells inside the particular cell is considered to minimize the induced error more accurately. This basic assumption is intuitively obvious, and may be analytically proved for the following classical case.

Consider transient radial flow through a homogeneous, unconfined aquifer. We get the equation, Jacob, [1950]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[rh\frac{\partial h}{\partial r}\right] = S/k\,\frac{\partial h}{\partial t} \tag{1.9}$$

where h is hydraulic head, r the radial coordinate, S the storage coefficient, and k the hydraulic conductivity. Let Q be a constant (positive) well production at the origin. Initial and boundary conditions are:

$$\begin{aligned} \lim_{t\to 0} \quad h(r,t) &= h_o \\ \lim_{r\to\infty} \quad h(r,t) &= h_o \\ \lim_{r\to 0^+} \quad r\,\frac{\partial h^2}{\partial r} &= \frac{Q}{\pi k} \end{aligned} \tag{1.10}$$

where h_o is the initial hydraulic head in the aquifer.

Haimes et al, [1968], show that if drawdowns are small compared with the aquifer thickness, transmissivity coefficient is defined $T = k\bar{h}$ where $\bar{h}$ is the mean value of h, and the solution to (1.9) subject to (1.10) is:

$$h = h_o + \frac{Q}{4\pi T}\,E_i\left(-\frac{sr^2}{4Tt}\right), \qquad E_i(x) \triangleq \int_x^\infty -\frac{e^{-u}}{u}\,du \tag{1.11}$$

A sensitivity analysis for that case may be done to determine the sensitivity of the solution to certain parameters. Rewrite Equation (1.11):

$$h = h_o + \frac{Q}{4\pi T}\left(- \lim_{A\to\infty} \int_{\frac{sr^2}{4Tt}}^{A} \frac{e^{-u}}{u}\, d\, u\right) \tag{1.12}$$

Through aggregating pumpage from different wells at a single point (the multicell model principle) we in fact are changing the variable r , which is the distance from the origin. The sensitivity of the solution h to perturbations in r is approximated by the following equation:

$$\begin{aligned} \frac{\partial h}{\partial r} &= \frac{Q}{4\pi T} \cdot \left[\frac{Sr}{2Tt} \; e^{-Sr^2/4Tt} \Big/ \frac{Sr^2}{4Tt}\right] \\ &= \frac{Q}{2\pi Tr}\, e^{-\frac{Sr^2}{4Tt}} = \frac{C_1}{re^{C_2 r^2}} \end{aligned} \tag{1.13}$$

where

$$C_1 = \frac{Q}{2\pi T} \qquad\qquad C_2 = \frac{S}{4Tt}$$

The effect of perturbating r by δr on the computed head h at a point located at a distance r may be approximated as:

$$\delta h \cong \frac{C_1}{re^{C_2 r^2}}\, \delta r \tag{1.14}$$

It is evident, that as the distance r between the pumping well and the measuring point is larger, the error caused in computing the drawdown at $r \pm \delta r$ is reduced, and is approximated by the expression (1.14).

Such a sensitivity analysis, if performed for more complex systems which are nonhomogeneous with irregular boundaries, is expected to be more tedious if possible at all. Later in this study, application of the proposed procedure to the real system case study shows induction of negligible error due to the superposition technique as compared with a much more detailed one. Furthermore the modeling efforts are considerably easier.

1.5.2 The Uniqueness of the Decomposition Approach Solution

Given a system which may be described by a set of partial differential equations and the associated boundary and initial conditions, the solution strategy basically suggested in this study is as follows:

1. Solve the system equations (via the multicell model).
2. Use the solution to compute boundary conditions for a particular subsystem (particular cell).
3. Solve the particular cell model. This solution is subject to the boundary conditions derived from the multicell model. This solution is applied to solve for the system response inside the cell.

Dealing with the problem of flow in a porous media, the mathematical model used for describing the system is comprised of the diffusion equation, namely partial differential equation of the parabolic type, Bear, [1972].

Consider the one-dimensional operator L: $Ly = 0$ (1.15)

where

$$L = \frac{\partial}{\partial t}(\cdot) - D\frac{\partial^2}{\partial x^2}(\cdot) \qquad \begin{array}{l} x \varepsilon [0,1] \\ t \varepsilon [0,T] \end{array} \tag{1.16}$$

and boundary conditions:

$$y(x,0) = g(x) \tag{1.17}$$

$$y(0,t) = y(1,t) = 0 \tag{1.18}$$

The solution for this case is explicitly known to be (Roach, [1970]:

$$y(x,t) = \sum_{i=1}^{\infty} \exp[-D(i\pi)^2 t]\cdot[\int_0^1 g(x) \sin i\pi x \, dx] \cdot \sin i\pi x \tag{1.19}$$

Assume now that the solution (1.19) is used to specify the value of y corresponding to the values of the spatial variable $x = a$, $x = b$ such that $0 < a < b < 1$.

$$\begin{aligned} y(a,t) &= y_1(a,t) = h_1(t) \\ y(b,t) &= y_2(b,t) = h_2(t) \end{aligned} \tag{1.20}$$

A particular problem for $x \varepsilon [a,b]$ is now performed. We now consider the operator L': $L' y_p = 0$ (1.21)

where

$$L' = \frac{\partial}{\partial t}(\cdot) - D\frac{\partial^2}{x^2}(\cdot) \qquad x \varepsilon [a,b], \quad t \varepsilon [0,\infty] \tag{1.22}$$

and boundary conditions:

$$y_p(x,0) \;=\; g(x) \tag{1.23}$$

$$y_p(a,t) \;=\; h_1(t) \tag{1.24}$$

$$y_p(b,t) \;=\; h_2(t) \tag{1.25}$$

The solution for the problem stated in (1.21) - (1.25) is

$$y_p(x,t) \;=\; f_p(x,t) \qquad x \in [a,b] \quad t \in [0,\infty] \tag{1.26}$$

(1.26) is assumed to pertain to a unique solution for operator L' and the associated boundary conditions.

The procedure stated at the beginning of this discussion (1) - (3), is essentially illustrated through the derivations in (1.15) - (1.26).

THEOREM: The solution $y(x,t)$ in (1.19) is identical to the solution $y_p(x,t)$ in (1.26) for all $x \in [a,b]$, $t \in [0,\infty]$ if and only if $y(x,t)$ is a unique solution of operator L and $y_p(x,t)$ is a unique solution of operator L'.

PROOF: Let Z_1, Z_p be two distinct solutions for (1.19) and (1.26), respectively, $x \in [a,b]$.

define $\quad Z \;=\; Z_1 - Z_p \quad$ (1.27)

$$L^2 \;=\; \frac{\partial}{\partial t}(\cdot) - \frac{\partial^2}{\partial x^2}(\cdot) \qquad x \in [a,b] \qquad t \in [0,\infty] \tag{1.28}$$

$$L^2 Z = L^2(Z_1 - Z_p) = L^2 Z_1 - L^2 Z_p$$

$$= \left(\frac{\partial}{\partial t} Z_1 - \frac{\partial^2}{\partial x^2} Z_1\right) - \left(\frac{\partial}{\partial t} Z_p - \frac{\partial^2}{\partial x^2} Z_p\right)$$

$$= 0 - 0 = 0 \qquad (1.29)$$

$$Z(x,0) = Z_1(x,0) - Z_p(x,0) = g(x) - g(x) = 0 \qquad (1.30)$$

$$Z(a,t) = Z_1(a,t) - Z_p(a,t) = h_1(t) - h_1(t) = 0 \qquad (1.31)$$

$$Z(b,t) = Z_1(b,t) - Z_p(b,t) = h_2(t) - h_2(t) = 0 \qquad (1.32)$$

(1.29) - (1.32) hold true provided both Z_1 and Z_p each constitute a unique solution for L and L', respectively.

Equations (1.27) - (1.32) constitute a problem whose solution is $Z(x,t) = 0 \quad \forall\, x,t$, Mikhlin, [1970], and consequently

$$Z_1(x,t) = Z_p(x,t) \qquad x \in [a,b] \qquad t \in [0,\infty] \qquad (1.33)$$

To conclude this part of our discussion, the multicell-particular cell modeling technique approximates the unique solution for the drawdown distribution provided both mathematical models each constitute a unique solution.

The hierarchy of groundwater simulation models (Figure 1.1) is based on the analytical groundwork which the previous discussion provides. Thus, we first solve the multicell simulation model. This model will serve as the higher level in the simulation hierarchy.

Consequently, we have the particular cell model solution lower in the hierarchy. The higher level provides the lower level with boundary flow equations which in turn are used in the particular cell model formulation to specify the "rest of the world" effect on the modeled subarea. The procedure described here was applied to the case study as discussed and summarized in Phase I.

A most appreciable advantage of the proposed procedure is that the digital computer time consumed is small. In order to determine 10 years' drawdown at wells located in a particular area (Cell 4), Maddock's groundwater simulation program, Maddock, [1969], on the UNIVAC 1108 consumed 59 seconds to simulate the overall aquifer system in one single stage. The two-stage simulation, however, consumed less than 14 seconds, of which the particular cell simulation (with Maddock's program) consumed 10 seconds.

CHAPTER 2

IDENTIFICATION OF GROUNDWATER PARAMETERS IN A MULTICELL SYSTEM

2.1 INTRODUCTION

Groundwater is a vital source of water supply. Its wise management presents numerous problems of varying degrees of complexity. Thus a broad approach is required to analyze and solve these problems. One of the problems is that there are not enough data available on the system being modeled. Thus water resources systems analysts develop a nonrepresentative model of the system, which often results in an erroneous output from the model. This chapter is concerned with developing a reasonably representative model of a groundwater system, using additional information so that a model output with a high degree of accuracy can be obtained. Hence, in the process of evaluating groundwater as a continuous source of water supply, the analyst may consider the following questions:

(1) What system model has to be built in order to closely represent the real system?

(2) What are the errors involved in modeling?

(3) What are the effects of model errors on the output of predicted water levels?

The purpose of this chapter is to answer the above fundamental and important questions faced in modeling a groundwater system.
Attention is primarily directed toward a sensitivity analysis of identifying parameters of confined aquifer models.

2.1.1 Motivation

Identification of unknown aquifer parameters is essential for making optimal decisions in the planning of a water resources system where groundwater or the conjunctive effect of ground and surface hydrology is considered. Obtaining the required aquifer system parameter values directly by an extensive observation system would be very difficult. For this reason most of the parameter values used are deduced from the behavior of the real system rather than from direct observation. Mathematical models which approximate a real system play an important part in this regard. The basic motivation of this chapter is to identify the unknown parameters so that the mathematical model closely represents the real system response.

Applying this motivation to this phase of the project accomplishes the following:

(1) it develops a drawdown forecast model.

(2) it analyzes sensitivity of computed head values to systematic changes in different model parameters.

(3) it uses the Fairfield-New Baltimore area in Southern Ohio as a case study.

2.1.2 Objective

The main objective can be described as follows:

(1) To develop an efficient means of identifying the parameter of an aquifer system that is confined, unconfined (when drawdown is small compared to the saturated thickness) or both, using additional information so that the model becomes less sensitive to error in parameter identification. To do this, the aquifer is decomposed into blocks known as cells according to available hydrological and other information. A set of difference equations is established for particular cells based on the interflow between adjacent cells. To obtain an accurate estimate of drawdown at a given point of interest, one can isolate the cell in which the point of interest is located. This cell may then be modeled in greater detail, using a mathematical model which considers the particular boundary conditions related to the adjacent cells as a function of time. This decomposition approach uses much more available information than any other approach developed for identifying aquifer parameters in groundwater systems.

(2) To show that the above decomposition approach to parameter identification for predicting drawdown of groundwater systems yields better results than earlier work in this area. Note that earlier parameter identification (presented in Phase I and II) considers (i) to be the whole aquifer as a single cell and (ii), the transmissivity, to be spatially distributed in two-dimensional coordinates.

The scope of the following is limited to these assumptions:

(1) The aquifer model can be described by a linear parabolic partial differential equation.

(2) Transmissivity is decomposed on a two-dimensional space.

(3) Storage coefficient as well as the initial and boundary conditions of the aquifer, together with the recharge and withdrawal, are known.

2.1.3 Literature Survey

Practical water resources problems are governed by partial differential equations containing a number of physical parameters. These unknown parameters are usually determined empirically. However, over the past several years, investigators have presented theoretical ways of identifying them from data observed in the field. Thus the theoretical ways of identifying these parameters are equivalent to the problem of parameter identification of a partial differential equation. This area is not well developed and many problems remain unsolved as yet. The problem stems from the fact that the theory of partial differential equations is complex and difficult to apply. Most partial differential equations of interest in engineering have no analytical solutions, and the existing numerical techniques to solve them are not completely satisfactory.

For identification of partial differential equations, most techniques focus on identifying a constant parameter in a one-dimensional system, whereas this chapter focuses on identifying varying parameters in a multidimensional system. The literature dealing with parameter identification in unsteady groundwater flow governed by a partial differential equation is widespread.

To the problem of water resources analysis, Yeh and Tauxe (Yeh and Tauxe, 1971) applied quasi-linearization in identifying the parameters of a homogeneous and isotropic confined aquifer system. A further extension of this model to a finite leaky aquifer system was studied by Marino and Yeh (Marino and Yeh, 1973). The major criticism of quasilinearization is its small region of convergence. Also, for systems of more than one dimension, it produces large sets of ordinary differential equations which are obtained by transforming partial differential equations, thus increasing considerably the problem's dimensionality.

For a particular identification of aquifer parameters, Haimes, et al, [1968], applied decomposition and multilevel optimization techniques where the aquifer system model is decomposed into a set of independent subsystems each of which is described by a one-dimensional, constant-parameter partial differential equation. This approach is appealing for its relative simplicity. However, it cannot handle complex boundary characteristics which cause interference with well response, since the image equations (which describe interations among subsystems) become rather complicated. Also, variable recharge produced by lakes and/or rivers cannot be handled, since the input-output water balance of the aquifer is assumed constant (indeed, the computational simplicity of the method would be spoiled since no analytical solution for the subsystems' equations exists for the case variable). Other comments on this approach can be found in Birkhoff and Varga (Birkhoff and Varga, 1959). In this chapter, both complex boundaries and recharge patterns can be handled with the scheme developed in section 2.2.

Falkenbarg (Falkenbarg, 1971) identified variable parameter one-dimensional equations by transforming the partial differential equation into an integral equation representation. Using a functional approach, he generates an approximate solution for the distributed system, using the integral equation. This approximate solution is then used to identify the equation parameters on a least-square basis. Extensions of this methodology to handle two-dimensional partial differential equations has not been done up to now and therefore cannot be applied here.

Kleinecke (Kleinecke, 1971) transforms the partial differential equation into a set of difference equations, and using an equation balance error criterion, formulates the aquifer model calibration problem as a linear programming problem. The validity of this approach has been questioned because of the difficulty of accurately estimating time and spatial derivatives using discrete data on the function being identified. The approach in general seems to be very sensitive to the level of measurement error and therefore of little use.

Karplus and Kawamoto (Karplus and Kawamato, 1966) apply sensitivity analysis to identify constant parameters in a multidimensional partial differential equation. Senfield (Senfield, 1971) follows the same approach. The identification problem is posed as a minimization problem. Solution of the partial differential equation is required to match the measured response of the physical system. The parameters are identified on a least-squares basis using a steepest-descent method. The main drawback of this approach is the slow convergence rate of the steepest-descent method. This, combined with the number of sensitivity equations (equal to the parameters being identified) that have to be resolved at each iteration, may be an overburden from a computational viewpoint.

Phillipson (Phillipson, 1971) solves the problem of identifying initial and boundary conditions for systems described by linear parabolic and second-order hyperbolic partial differential equations. He casts the problem within a variational framework and characterizes extremals of quadratic functionals constrained by a partial differential equation by applying known results from the theory of optimal control of distributed parameter systems developed by Lions (lions, 1971).

In Phase I we formulate the identification problem using steps similar to those of Phillipson (Phillipson, 1972). On the other hand, we use Lions (Lions, 1971) for solving the quadratic approximation of the parameter identification as a variational problem.

The different methodologies of identifying parameters mentioned above have some features in common -- they all primarily assume an aquifer either as a single cell or as a one-dimensional flow system or both. These assumptions have the following problems:

(1) Considering an aquifer as a single cell leads to assuming a homogeneous property of the aquifer. In the real world, the discontinuity of soil characteristics in an aquifer causes the aquifer to have non-homogeneous properties. Hence the assumption of homogeneity is erroneous.

(2) Groundwater flow is multidimensional. Hence the assumption of one-dimensional flow becomes nonrepresentative of the actual groundwater flow.

In general, errors associated with mathematical assumptions results from using a relatively simple mathematical expression to represent a complex natural physical system. To cope with this problem reasonably, this chapter implements a better procedure for groundwater system modeling. In this procedure the whole aquifer is decomposed into different cells, taking into account the fact that interflow between adjacent cells results in a set of difference equations. In chapter 1 this procedure is discussed.

To identify the parameter (transmissivity) of a particular cell, the cell is modeled in greater detail and calibrated via Marquardt's Non-linear Algorithm (Marquardt, 1963).

Consequently in this approach, by decomposing the system and considering multidimensional flow, we assign more importance to the nonhomogeneous soil characteristics and the two-dimensional flow pattern of an aquifer. Finally, additional information generated due to disintegration of the aquifer system leads to a parameter identification procedure which results in a less sensitive output, even if some error exists in basic input information.

2.1.4 Aquifer Identification Problem

Using the models described in Chapter 1, Equations (1.1) and (1.7), to forcast aquifer water levels, the following information for each cell should be obtained:

1. Length of the perpendicular sector associated with the segment between cells, W
2. Distance between centers of cells, L
3. Hydraulic conductivity averaged between cells, K
4. Effective aquifer depth averaged between cells, E
5. Elevation at the top of the aquifer averaged between cells, F
6. Water elevation, h
7. Forcing function or pumpage, Q
8. Storage function, S
9. Transmissivity function, T
10. Initial conditions
11. Boundary conditions

Determining the above eleven types of input data or parameters comprises the aquifer system identification problem, and identifying each of these parameters is difficult. For example, identifying Q requires determining the pumpage and recharge pattern, rain infiltration, river and lake percolation, and leakage and losses to make a water balance of the total water input to the aquifer. A similar puzzle is determining the aquifer's initial and boundary conditions (I.C. & B.C.). This is known as a state identification problem. Transmissivity and storativity are highly variable discretely distributed parameters. This is due to the wide variety of geological materials and structures an aquifer can be composed of. Such characteristics pose serious problems in identifying aquifer model transmissivity and storativity. In general the eleven points mentioned above are related to each other and can be considered a single problem composed of many subproblems. This chapter addresses itself to a single subproblem: Identifying the particular cell transmissivity function using more hydrological and geological information. It is assumed that pumpage, elevation, storage function, conductivity, I.C. and B.C. are already known. The problem can be stated as follows:

Given the following information on each cell

(1) initial and boundary conditions

(2) storage coefficient

(3) conductivity

(4) well pumpage records

(5) water elevation records

(6) topology

estimate the value of T (model transmissivity function) on the basis of the above information, using some curve-fitting criterion. Some factors which complicate the solution to this problem, are:

1. Since the aquifer water sources are random variables, it is difficult to estimate accurately the input function (Q) of each cell.
2. As it is not feasible to collect data for an entire particular-cell, crude discretely distributed data are used to estimate the overall distributed parameter function of a cell.
3. It is difficult to determine initial conditions, boundary conditions and topology of each cell.

2.1.5 Aquifer System Identification

Due to the heterogeneous property of most aquifers, the assumption that the groundwater system has distributed rather than lumped parameters is inherently more realistic. In this regard, two basically different approaches may be used to get useful representations for the heterogeneous properties of the present system. One approach is to subdivide the aquifer into a finite number of areas of specified geometry, each of which is assumed to be homogeneous with respect to transmissivity and storage. The simplest such case is the analysis of a lumped system for which the entire aquifer is considered to have homogeneous transmissivity and storage. The second approach is to define aquifer properties through a functional relationship which provides spatial variation. In this chapter a mixed approach of the above two methods will be considered. The whole aquifer is subdivided into a finite number of blocks known as particular cells, each of which has

(1) Constant storativity and

(2) Spatially distributed transmissivity.

Thus the identification problem in groundwater hydrology involves determining the distribution of parameters which characterize a particular cell from observations of pumping and recharge rates, flow at boundaries, water levels, and topology.

In order to predict future system response of a particular cell using equation (2.3), one should know the following about each cell:

(1) Boundary conditions including additional interflow information between cells obtained from multicell model equation (2.2).

(2) Production rates (i.e., rate of pumping, Q).

(3) Values of T and S.

It is easy to obtain the first two pices of information from observed data at specified locations, whereas collecting data for (3) creates a problem since no detailed knowledge of the variation of $T(x,y)$ and $S(x,y)$ is available. One way to handle this is to formulate an inverse problem. Thus, utilizing the observed information as input, an inverse problem in the aquifer system can be formed:

Given some function

$$F(h - \hat{h})$$

where $\hat{h}$ = observed head &

$h = h(T,S)$ = calculated head

How must T and S be chosen so that F is minimized? An answer to this question enables one to predict accurately the system response to future modes of operation. So it can be assumed a useful description of the system is given by specifying T and S which will minimize an appropriate criterion function.

2.2 IDENTIFICATION PROBLEM

2.2.1 Introduction

The important step in the identification of a parameter problem is to choose the model topology for the system being considered. In addition, one will need to determine the existence and iniqueness of a solution to the model and to have the capability of solving the equations governing it. Selecting the model for the aquifer has already been discussed in Chapter 1. The next step, developing an identification algorithm for model identification, is the main topic of this Chapter.

2.2.2 Composition of the Identification Problem

As mentioned in the last chapter, the mathematical model of the present system consists of two parts

(1) multicell model

(2) particular cell model

2.2.2.1 *Multicell model contribution for parameter identification problem*

The multicell model described by equation (1.7) is repeated for convenience below

$$\sum_j \frac{W_{ji}C_{ji}}{L_{ji}}(h_{jm} - h_{im}) - \frac{W_{ji}K_{ji}}{2L_{ji}}[(h_{jm})^2 - (h_{im})^2]$$
$$= \frac{A_i S_i}{\Delta t}(h_{i,m+1} - h_{i,m}) - Q_{i,m} \qquad (2.1)$$

The flow between the j-th cell and the ith cell during time period m is:

$$\sum_j \frac{W_{ji}C_{ji}}{L_{ji}}(h_{jm} - h_{im}) - \frac{W_{ji}K_{ji}}{2L_{ji}}[(h_{jm})^2 - (h_{im})^2] \qquad (2.2)$$

2.2.2.2 *Particularcell Parameter Identification*

For the particularcell, a more detailed formulation is used, and the above computed flow (2.2) is then distributed along the boundary line according to spatial and hydrological considerations.

The particularcell model under consideration as described by equation (1.1) can be written more specifically for cell j as

$$\frac{\partial}{\partial x}(T_j \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_j \frac{\partial h}{\partial y}) = S_j \frac{\partial h}{\partial t} + Q_j \qquad (2.3)$$

$$h_j(x,y,o) = h_{oj} \qquad (2.4)$$

$$\frac{\partial h}{\partial \vec{n}} \mid r_i = o_j \quad h_j(x,y,t) \mid r_2 = h_{1j} \qquad (2.5)$$

$$Q_j(x,y,t) \in R_j \qquad (2.6)$$

where $h_j(x,y,t)$ = drawdown at location (x,y) of cell j and time t. $Q_j(x,y,t)$ = net discharge rate per unit area, including recharge, leakage etc. at location (x,y) of cell j and time t. The initial and boundary conditions of the system are respectively given by (2.4) and (2.5), r_1 and r_2 denote the boundary geometry, R_j in equation (2.6) is the domain of (2.4) - (2.5).

The model described in (2.3) - (2.6) is not completely determined because the function $T_j(x,y)$ is unknown; therefore, the question arises as how to determine $T_j(x,y)$. The identification of the function $T_j(x,y)$ for a particular cell is known as a parameter identification, system identification, parameter estimation or model calibration.

Since the transmissivity value, $T_j(x,y)$ is not known, the response $h_j(x,y,t)$ cannot be computed from (2.3) - (2.6). The identification problem is to estimate the value of the transmissivity function $T_j(x,y)$, so that a specified performance criterion is satisfied. Choosing a performance criterion however, depends on many factors, including, for example, the model representing the physical system, the number of data points, the sensitivity of parameters as related to performance function, etc. A least-square norm of the output error, i.e., between observed and calculated values for the water head, is selected as the performance function.

This function $J_j(T(x,y))$ is expressed as

$$J_j(T(x,y)) = \int_0^t \int_{\Omega j} [h_j(x,y,t;T) - \hat{h}_j(x,y,t)]^2 \, dt \, d\Omega_j \quad (2.7)$$

where

Ω_j = the area of cell j

$h_j(x,y,t,T)$ = the model output for a given function $T_j(x,y)$

$H_j(x,y,t)$ = the observed value of the waterhead of various points in space and time over the area of cell j

Complete knowledge of a specific cell's geology is required to determine the mathematical structure of $T_j(x,y)$. The difficulties involved in determining transmissivity from physical measurements force hydrologists to pursue indirect methods. Accordingly, a scond-order polynomial representation of transmissivity function is utilized. The representation of transmissivity as a linear function in spatial coordinates was originally developed in Phase I, then it was modified to a second order polynomial in Phase II. The second-order polynomial representation of $T_j(x,y)$ which belongs to the space of positive polynomials in x and y is

$$T_j(x,y) = b_1 x^2 + b_2 y^2 + b_3 x + b_4 y + b_5 \quad (2.8)$$

where b_1, b_2, b_3, b_4 and b_5 are unknown coefficients to be estimated.

The identification problem can now be stated as follows:

$$\text{Minimize } J_j(T_j(x,y)) = \text{Min} \left\{ \int_o^t \int_{\Omega j} [h_j(x,y,t,T) - \hat{h}_j(x,y,t)]^2 \, dt \, d\Omega j \right\} \tag{2.9}$$

Subject to the constraints set

$$\left.\begin{array}{l} \frac{\partial}{\partial x}(T_j(x,y) \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(T_j(x,y) \frac{\partial h}{\partial y} = S_j \frac{\partial h}{\partial t} + Q_j(x,y,t) \\ \quad h_j(x,y,o) = h_o \\ \frac{h}{\partial \vec{n}} \Big| r_i = o_j \; ; \; h_j(x,y,t) \Big| r_2 = h_1 \\ \quad Q_j(x,y,t) \; \varepsilon \; R_j \end{array}\right\} \tag{2.10}$$

The search for a transmissivity function $T_j(x,y)$ which minimizes the objective function (2.9) constitutes the identification algorithm for a particular cell. The Marquardt Algorithm for least-squares estimation of nonlinear parameters (lopez, 1973) as used for parameter identification is found to be an effective approach in this regard.

Once the parameters (b_1, b_2, b_3, b_4 & b_5) representing spatially distributed transmissivity function $T_j(x,y)$ of cell j is identified, the next task will be to find the average value of tansmissivity for cell j-$T_{j\ av}$ as follows:

$$T_{j\ av} = \frac{\int_x \int_y T_j(x,y)\ dx\ dy}{\int_x \int_y dx\ dy} \tag{2.11}$$

where $\int_x \int_y T_j(x,y)\ dx\ dy$

is the sum of transmissivities at different points over the entire particularcell j

and $\int_x \int_y dx\ dy$ is the total area of cell j

2.2.3 Iterative Procedure for Identification Problem

Consider a number of cells constituting an aquifer. It is assumed that within the times considered there is no change in the aquifer's boundary conditions. Thus based on geohydrological considerations, a two-dimensional system model comprised of cells can be formed. Water in adjacent cells can flow from one to another. Hence for an n-cell aquifer system, the following approach is proposed as a solution to the identification problem:

(1) Make an initial guess for the vector $\underset{\sim}{T}_{av}$

$$\underset{\sim}{T}^{o}_{av} = \begin{bmatrix} T^{o}_{1_{av}} \\ T^{o}_{2_{av}} \\ T^{o}_{i_{av}} \\ T^{o}_{j_{av}} \\ T^{o}_{n_{av}} \end{bmatrix} \tag{2.12}$$

(2) Substitute T in the relation

$$K_{ji} = \frac{T_{ji}}{D_{ji}} \tag{2.13}$$

where K_{ji} = conductivity averaged between cells j and i

$T_{ji} = \dfrac{T_{j\ av} + T_{i\ av}}{2}$ = transmissivity averaged between cells j and i

D_{ji} = flow depth averaged between cells j and i

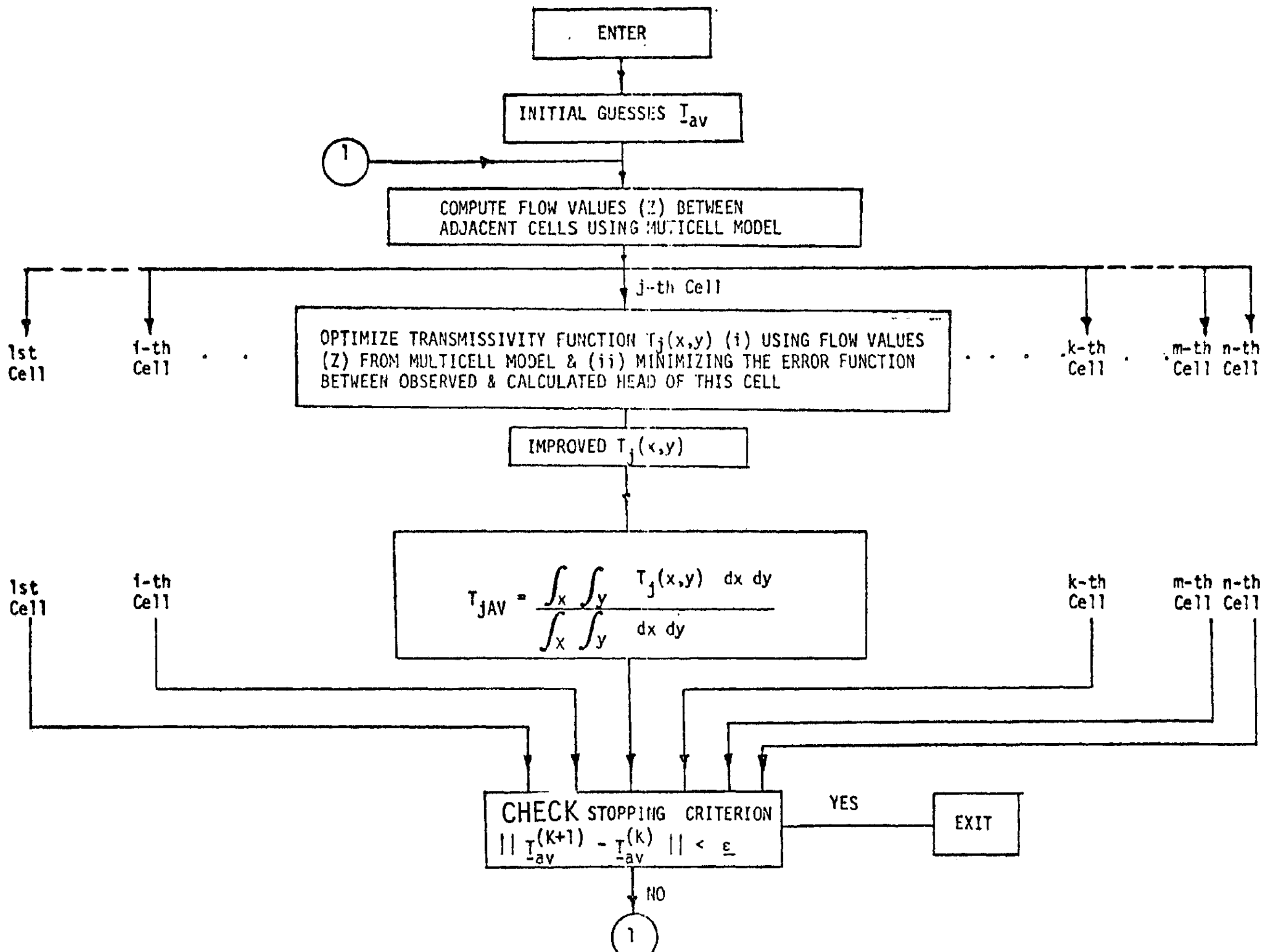
to get the conductivity K_{ji}

(3) Solve multicell model equation (2.1) to compute flow values between adjacent cells and water head at different times. To do so, use the information generated in step (2) above.

(4) Optimize transmissivity function T(x,y) for each particular cell by minimizing the error function between observed and calculated values of drawdown at specified points for each cell. Calculated values of drawdown are subject to flow values of multicell model equation (2.1).

(5) Transform improved $T(x,y)$ of step (4) into average transmissivity T_{av} using equation (2.11) - for each cell.

(6) Compare the average transmissivity vector $\underline{T}_{av}$ obtained in step (5) with the initial guess of $\underline{T}_{av}$ in step (1). If this difference is less than a vector of convergence factor $\underline{\varepsilon}$, then stop the procedure. Otherwise go to step (1) (use improved $\underline{T}_{av}^{(K+1)}$ obtained in step (5) rather than initial guess $\underline{T}_{av}^{(K)}$). A flow diagram of the identification algorithm is depicted in Figure 2.1. The preceding theoretical concept was put on the Univac 1108 digital computer in fortran language to achieve our results.



BASIC SCHEME FOR THE ITERATIVE PROCESS

FIG. 2.1

2.3 CASE STUDY

2.3.1 Introduction

The purpose of this section is to illustrate the feasibility of the modeling technique proposed in the last chapter by means of a case study. The Fairfield-New Baltimore aquifer in the lower great Miami River Valley of southern Ohio is a typical example of a large water resources system. This example is well suited to testing the methodologies developed in this chapter. Even though the system is described in detail in Phases I and II, we represent it here for the completeness of the report.

2.3.2 Description of Real Aquifer System: Miami Conservency District

The area modeled for the validation of the identification algorithm is the Fairfield-New Baltimore area of the Miami Conservency District which consists of 32 square miles of the Great Miami River Valley southwest of Hamilton, Ohio. The area modeled possesses a sand and gravel aquifer that is bounded by the bedrock walls of the Great Miami River Valley. These walls form the boundary of the aquifer, with the exceptions of the west and the north, where the boundaries are arbitrary. For the west boundary the dry fork of the White Water River, located about two miles west of New Baltimore was selected. For the northern boundary a line through Fairfield near the southern city limit of Hamilton was chosen.

Geologically, the aquifer under study consists of glacial outwash, sands, and gravels of the Pleistocene Age. From the hydrogeological point of view, the aquifer area can be conveniently divided into three parts as follows:

In the central part of the area the aquifer material consists of stratified sand and gravel situated 150-200 feet below ground surface. Widely scattered lenses of clay and silt are also present but do not cover a sufficient area to cause any perceptible confining effects. In the southwest corner the sand and gravel is only about 80 feet thick.

Along the eastern edge of the area some three square miles consist of a sand and gravel aquifer which is about 100 to 150 feet thick and is overlain by about 100 feet of clay and silt.

In the western-most portion of the Fairfield-New Baltimore area, which covers about eight square miles, the aquifer is about 200 feet thick and is capped with a complex layer of till, silt and clay.

Groundwater is unconfined throughout most of the area. However, the mathematical condition that the drawdown be small as compared to the saturated thickness of the aquifer is satisfied. This condition permits use of the identification technique developed in this work.

The hydrologic and geologic characteristics of the Fairfield-New Baltimore aquifer have been extensively studied and a report [Spieker, 1968] provides an excellent source of information for the area.

2.3.2.1 *Estimation of the Input-Output Water Balance*

Concerning the hydrologic boundaries (i.e., boundary conditions), the aquifer is bounded by the vertical bedrock wall of the buried Miami Valley. The permeability of this rock is slight, yet it can contribute a significant amount of water to the system due to the very large contact area, therefore, a leakage boundary is introduced into the model. A second source of water is provided by the Great Miami River which traverses the aquifer as shown (Figure 4.3). The river strongly interfaces with the aquifer and is one of the most important components of the ground and surface water system.

The input-output water balance of the aquifer is made up of the following components:

(i) Recharging of Induced Steam Infiltration

This is a difficult system input to estimate. It is a highly variable quantity whose interaction with the aquifer depends on many factors, such as width and depth of the river, velocity of the streamflow, permeability of the streambed. The most critical of all these factors is the stream infiltration rate under conditions of low streamflow. Two estimates of this factor have been made for the area in question and, based on them, a range of 240,000 to 500,000 gpd per acre has been determined as the expected range of variation for the maximum infiltration rate all year round [Spieker, 1968]. Such a range indicates that the river is a large source of water for the aquifer; consequently, in the aquifer model the river has been modeled as a constant head boundary.

(ii) Recharge from Boundaries

The perimeter of the aquifer modeled is 220,000 feet, of which 180,000 feet are along the bedrock valley walls. The permeability has been estimated to be on the order of 1.5 gpd per sq. ft. These figures, when multiplied by the total area, yield 6.8 mgd coming from the bedrock formations into the aquifer. This last figure is used in this study.

(iii) Pumping

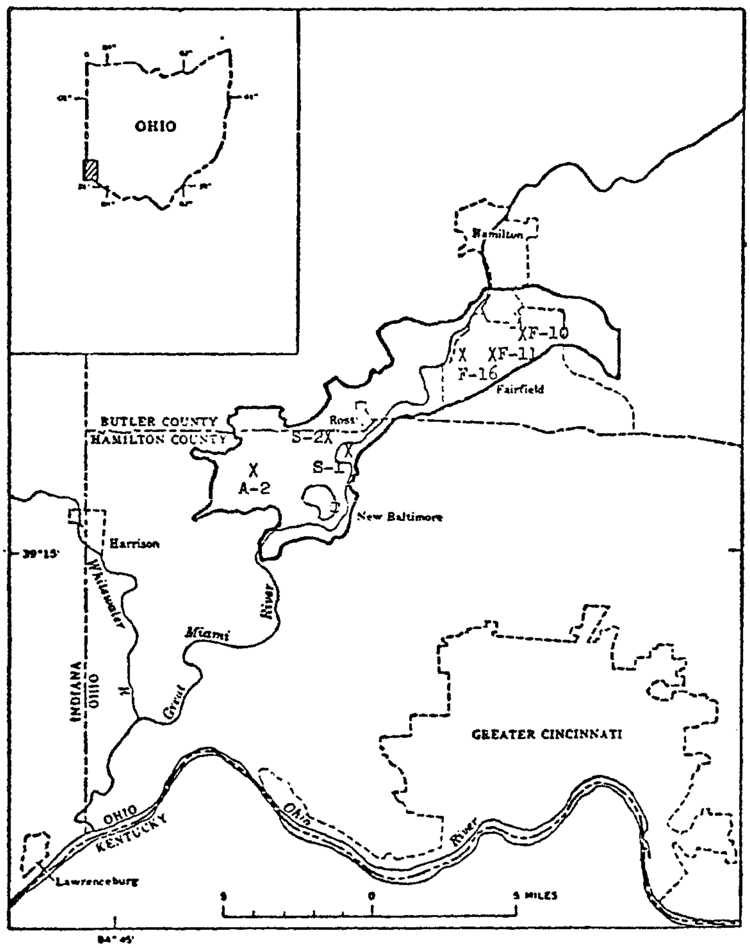
Pumping is concentrated in three well fields, namely, the well fields of Hamilton south (Fairfield), the Southwestern Ohio Water Co., and the U.S. Atomic Energy Commission. Pumping started in 1943 with eleven wells in Fairfield. These were operated from 1943 to 1945. Then, from 1945 to 1952 there was no significant pumping in the area. In 1952 Southwestern Co. installed a new well, S-1 (Figure 2.4). It was pumped from 1952 to 1955 at an average rate of 10 mgd. In 1955 a second well was installed, S-2 (Figure 2.4) The combined pumpage of S-1 and S-2 from 1955 to 1962 averaged 13.8 mgd. In 1956 the city of Hamilton installed a new well field (F-16, F-10, F-11) which was pumped from 1956 through 1962 at an average of 7.5 mgd. The U.S. Atomic Energy Commission well field A-2, has been pumped at an average of 1 mgd since 1952.

(iv) Initial Conditions

Records of water level in the area were not kept until pumping had started; therefore, it is difficult to determine the initial conditions of the system. Spieker [Spieker, 1968] estimated those conditions based on existing hydrographs of the area, present water level measurements, models' results, and river stages. In the present work, initial conditions for groundwater levels in the area were considered according to Speiker.

For the Fairfield-New Baltimore area only four reliable pumping tests have been performed to determine the aquifer transmissivity. Locations of test points are shown as T_1, T_2, T_3, T_4, (Figure 2.4). The storage coefficient has been considered based on Speiker.

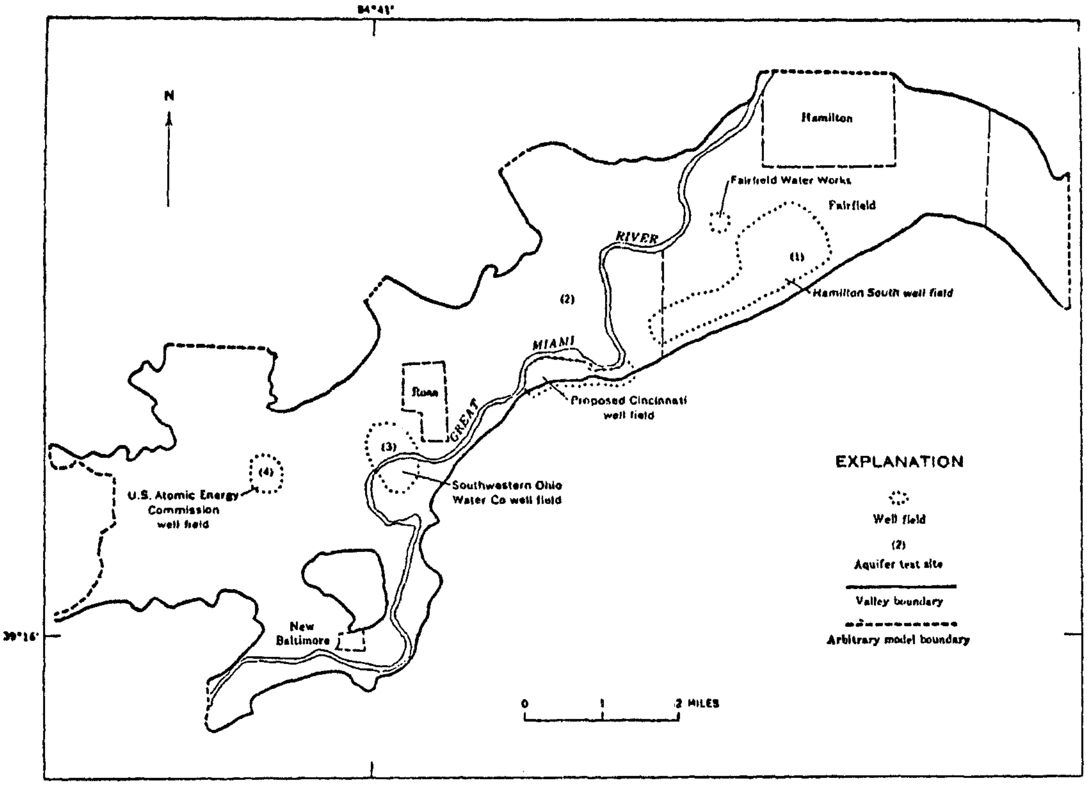
The construction and validation of an aquifer model for the Fairfield-New Baltimore area is an important step in this project since no prediction of the real system behavior can be made without such a component.



Ground Water In The Lower Great Miami River Valley, Ohio

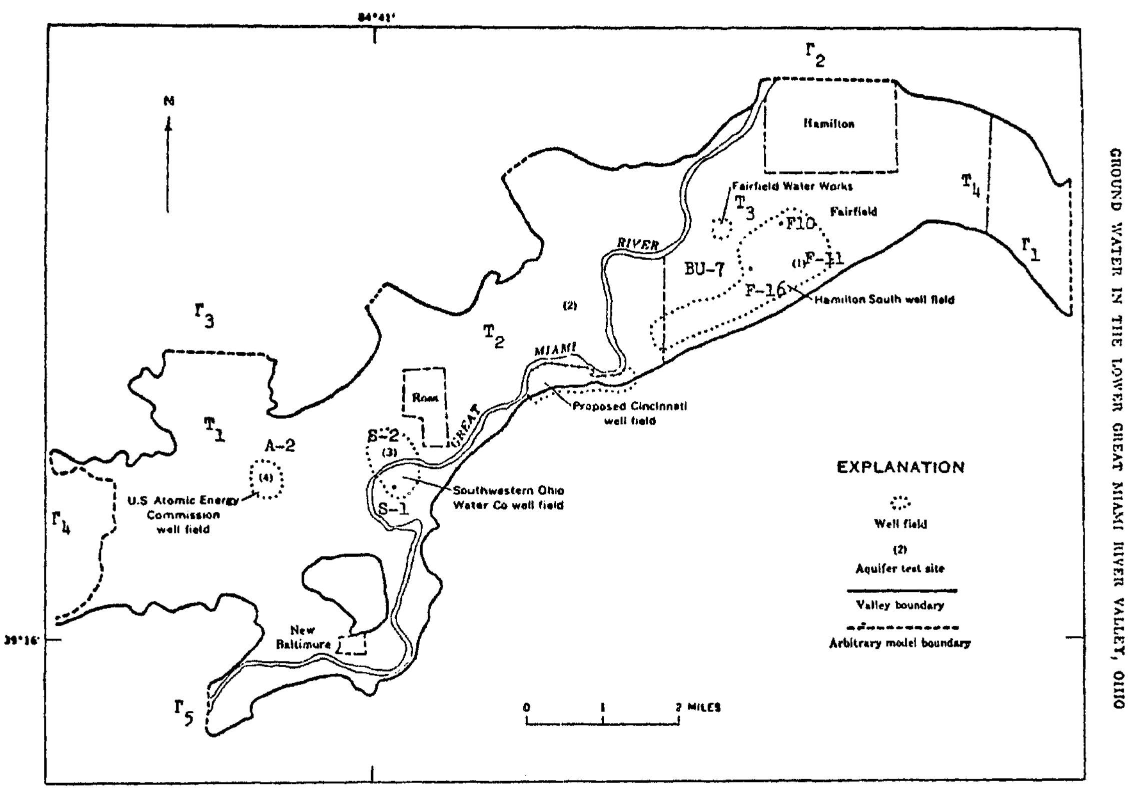
Fig. 2.2 - Location of the Fairfield-New Baltimore area, lower Great Miami River valley

Well Locations Marked (X)



DESCRIPTION OF THE LOWER GREAT MIAMI RIVER VALLEY, OHIO

FIGURE 2.3



—Location of existing well fields and of the proposed Cincinnati well field, Fairfield-New Baltimore area. Arbitrary limits of the modeled area, beyond which the aquifer extends, are indicated by dashed lines.

FIGURE 2.4

2.3.3 The Aquifer Model

The modeling of the real system described in the previous sections is described in this section. A computer program was written to simulate the aquifer. The system was divided into cells with differing characteristics (See Fig. 2 5). The data utilized include pumpage water elevations and cell boundary conditions and were taken from Spieker (Spieker, 1968). An explicit computation scheme can be used, if care is taken to avoid the stability problem by choosing an appropriately small time step. The semi-pervious bedrock which forms the natural boundaries for the groundwater system can be handled as part of the water balance of each cell (constant inflow). The river can be handled as constant head cells. Initial waterhead values in all cells are part of the input to the program. For each time period (one year) the forcing function (pumpage) at each cell is given.

The simulation model can produce two types of output:

(i) For each time period, the interflow between adjacent cells is provided.

(ii) For each time period the averaged water level is predicted in all cells.

Cells #4, #5 and #6 (See Fig. 2.5) were considered in this work due to the location of observation wells (F-10, F-11, F-16, S-1, S-2, and A-2) within these cells. Infiltration rates and the complete pumping history of these cells from 1952 to 1962, which were obtained from the Miami Conservancy District, are

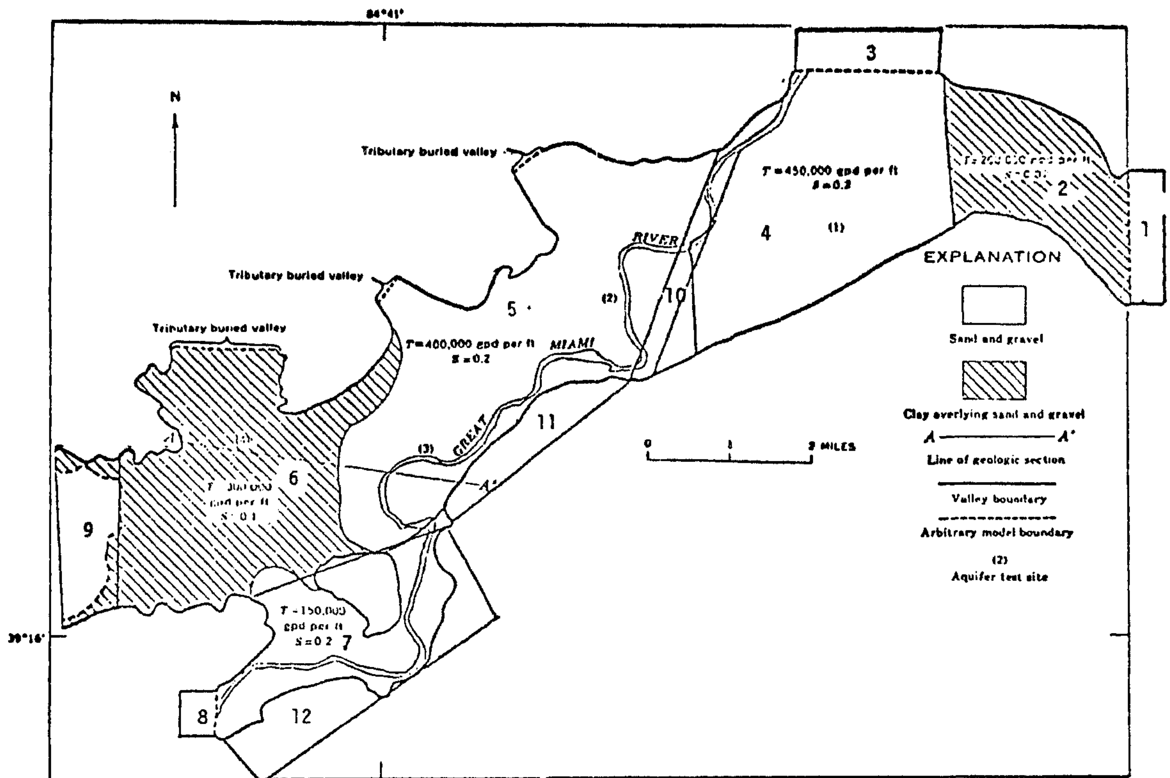


FIGURE 2.5

Analog Study of Increased Pumping Effects, Fairfield-New Baltimore Area

Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area. Cells assignment--Cells 10, 11, and 12 represent the river.

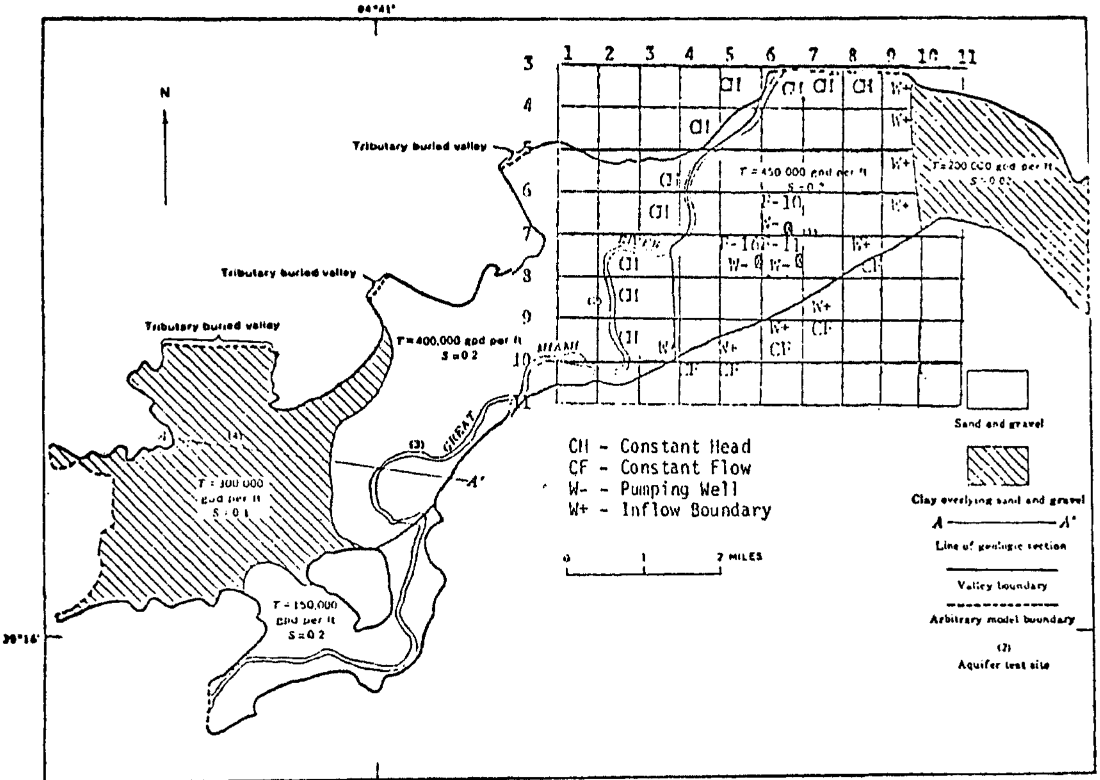
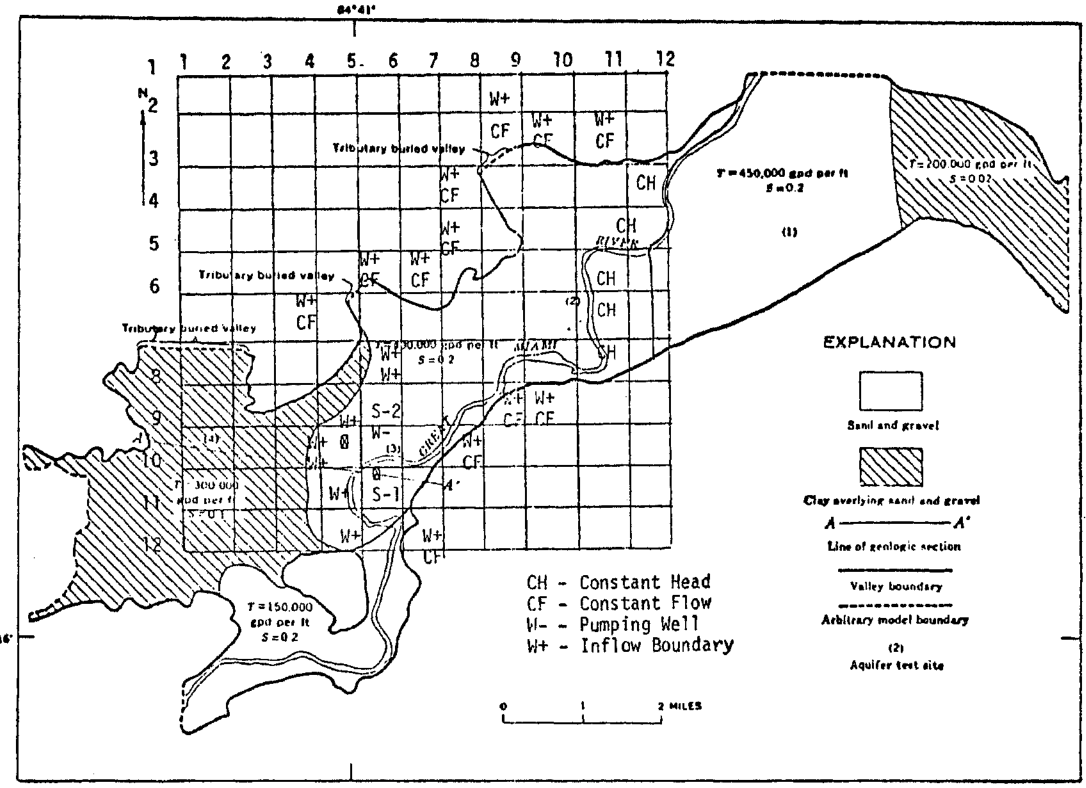


Figure 2.6 - Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area.

Cell #4 Discretization for the Detailed Modeling



Generalized geology and coefficients of transmissibility (*T*) and storage (*S*) of the Fairfield–New Baltimore area.

Cell #5 Discretization for the Detailed Modeling

FIGURE 2.7

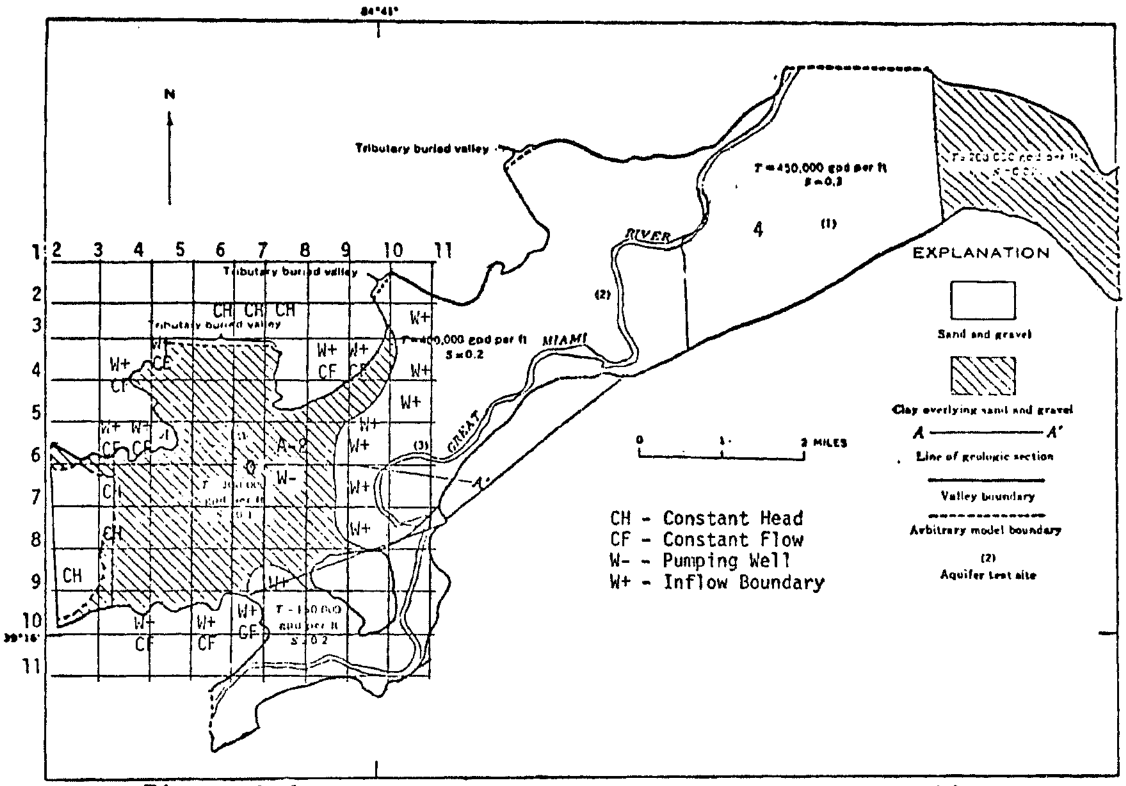


Figure 2.8 - Generalized geology and coefficients of transmissibility (T) and storage (S) of the Fairfield-New Baltimore area.

Cell #6 Discretization for the Detailed Modeling presented in Tables 2.1 and 2.2. A breakdown per month can be obtained from Speiker (1968). Location of the pumping wells is shown in Figure 2.4. Table 2.3 summarizes the characteristics of the cells under study. Figures 2.6, 2.7 and 2.8 indicate the constant head and recharging boundaries of the concerned cells.

To show the possible applications of the methodology developed in this chapter to the case under study, boundary conditions were taken for these three cells from the results of the multicell model. The method used for identifying the transmissivity function parameters of these cells is an iterative gradient algorithm developed by Lopez (Lopez, 1973) based on the maximum neighborhood method (Marquardt, 1963). Once the parameters defining the transmissivity function have been estimated, the appropriate next test of the calibrated equipment model is how well it predicts the aquifer's response to any demand placed on it.

2.3.4 Needs for Additional Information in Aquifer Modeling

The decomposition approach of aquifer modeling in this chapter stems from the intuition of developing an accurate groundwater model for great Miami River Basin using additional available information on the groundwater system. To do this, it is worthwhile to answer the following questions:

(1) What kind of modeling errors can we come up with in developing an accurate model?

(2) How can those errors be minimized?

Well Name	Cell Location	Pumping Periods 1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962
A-2	6	155	155	155	155	155	155	155	155	155	155	155
S-1 S-2	5	1512	1835	1762	2155	2031	2260	2019	2298	2223	2004	1951
F-10	4	0	0	500	0	338	377	381	372	356	354	357
F-11	4	0	500	0	0	423	471	477	465	445	443	446
F-16	4	500	0	0	500	338	377	381	372	356	354	357

TABLE 2.1

PUMPING HISTORY FAIRFIELD-NEW BALTIMORE AQUIFER. FIGURES ARE GIVEN IN FT^3/SEC. *100. DATA FROM 1958-62 WERE NOT USED IN THE IDENTIFICATION OF T

CELL #4	Boundary Points (I,J)	(7,8)	(7,9)	(8,7)	(9,4)	(9,5)	(9,6)			
		(See Fig. 4.5 for location of this coordinates								
	Infiltration Rate	5	5	5	5	5	5			
CELL #5	Boundary Points (I,J)	(3,11)	(3,10)	(3,9)	(3,8)	(4,8)	(5,8)	(6,7)	(6,6)	(6,4)
		(See Fig. 4.6 for location of this coordinates)								
	Infiltration Rate	18	12	12	12	12	12	12	12	6
	Boundary Points (I,J)	(6,5)	(7,4)	(8,10)	(8,9)	(9,7)	(9,8)	(10,6)	(11,6)	
	Infiltration Rate	6	6	6	6	6	6	12	6	
CELL #6	Boundary Points (I,J)	(4,5)	(5,5)	(6,4)	(6,5)	(3,8)	(4,8)	(4,9)		
		(See Fig. 4.7 for location of this coordinates)								
	Infiltration Rate	12	12	12	12	6	12	12		
	Boundary Points (I,J)	(3,10)	(9,4)	(9,5)	(9,6)	(9,7)	(9,8)			
	Infiltration Rate	6	12	12	12	12	2			

TABLE: 2.2 Infiltration Rates Fairfield-New Baltimore Aquifer (Units: ft^3/sec *100)

CHARACTERISTIC	CELL NO.	DESCRIPTION
Aquifer Type	4	Unconfined small marginal areas are of semi-confined type.
	5	
	6	
Storage Coefficient, s (Dimensionless)	4	0.2
	5	0.2
	6	0.1
Transmissivity Coefficient, T (ft/sec)	4	Unknown
	5	
	6	
Initial Head (in ft.)	4	552
	5	532
	6	524
Boundary Conditions	4	East & West: Inflows from Cell #2 & Const. Head North & South: Const. head & Constant flow
	5	East & West: Inflow from Cell #6 North & South Constant flow
	6	East & West: Inflow from Cell #5 & Constant head North & South: Constant flow
Wells	4	F-10, F-11, F-16
	5	S1 & S2
	6	A2
Approximate Area (in sq. miles)	4	7
	5	9
	6	8

TABLE: 2.3
AQUIFER DATA: FAIRFIELD-NEW BALTIMORE

As mentioned earlier, an error in groundwater modeling is defined as the absolute difference at a particular time between the waterhead computed at a given model location and the true water head at the corresponding location in the groundwater system:

$$E_{t,L} = ||h_{t,L} - \hat{h}_{t,L}|| \tag{2.14}$$

Where $E_{t,L}$ is the modeling error at location L (the L notation refers to the standard two-dimensional co-ordinate (x,y) system at time t; $h_{t,L}$ is the water level computed by the aquifer model at location L and time t and $\hat{h}_{t,L}$ is the true water level at a corresponding point and time in the groundwater system.

Modeling errors can be classified as those associated with:

(i) computation

(ii) mathematical assumption

(iii) basic data

Generally speaking, the three errors mentioned above include most of those in aquifer modeling. Our work was concerned with prediction errors caused by errors in basic data. We define an error in basic data as the difference between the estimated or measured value of a model variable and the corresponding true value of the groundwater system. Making errors in basic data is probably one of the major sources of errors in modeling.

Errors in basic data are classified as:

a) Errors in aquifer parameters

(i) coefficient of storage

(ii) coefficient of transmissivity

b) Errors in initial and final conditions of waterhead

c) Errors in input and output functions

(i) discharge (including pumpage)

(ii) recharge

d) Errors in boundary configuration

Each of the above includes some errors that lead to further errors in predicting future water levels.

Generally, data errors can be of several types, such as instrumental or measurement, interpolation sampling, and errors due to data not being representative of the aquifer. Measurement errors create minor problems whereas interpolation errors arise when field data are contoured to yeild estimates for all model nodes. Such contouring commonly is done for transmissivity and initial water levels. Sometimes field data may not be representative of or even from the aquifer being modeled. Measurements of water levels in wells affected by local pumping or in wells tapping parched water bodies, for example, will not be representative of aquifer conditions. Errors due to interpolation and nonrepresentative data are significant problems.

For the Miami River Basin in Southern Ohio the coefficient of storage is reasonably well known because adequate measurements of its value have been made over different sections of the aquifer. On the other hand, errors in estimates of transmissivity are present due to the consideration of its (transmissivity) average value over different sections of the aquifer. Finally the average value becomes nonrepresentative of that area due to its variation over space.

Error in initial water level may be due to

(i) measurement error

(ii) interpolation error

(iii) nonrepresentative location in the aquifer at that point in time.

In addition, errors in final water levels for one or more historical periods of time used in calibrating the model lead to modeling errors. Groundwater models commonly are calibrated by adjusting model parameters so that computed water levels match historically measured levels at one or more points in time. These final water levels can be in error for the same reasons that initial levels were in error.

Discharge and recharge estimates used in the model can be in error for several reasons, which can be classified as follows:

(i) errors in quantity

(ii) errors in the assumed location

(iii) errors related to time variations in discharge or recharge not accounted for by the model.

Much of the pumpage data in the Miami Basin are reasonably accurate as far as quantity and location of pumpage is concerned. Most of the recharge in the Miami Basin is caused by induced recharge from boundaries and subsurface flow from the Great Miami River. Adequate data from recharge are available from Speiker.

Errors also are introduced into the model because the model boundaries do not duplicate exactly those of the groundwater system.

The above study gives us some appreciation of different errors involved in groundwater modeling. Later we show by statistical analysis how data errors on transmissivity, storativity, pumpage and water head observation affect the groundwater model output.

2.4 COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSES FOR DECOMPOSED MODEL

2.4.1 Introduction

In this chapter the numerical methods used to accomplish the goals stated in previous chapters will be presented. As an example of using the identification algorithm developed in this chapter to estimate transmissivity values, the Fairfield-New Baltimore aquifer system is considered. The model-estimated parameters for transmissivity functions were then used for model validation to establish the capability of the model to predict real system behavior. This aquifer system was also used previously as a source for hydrogeological data for identifying and validating the model developed in Phases I and II. This facilitates a direct comparison of the results of this work with those models.

The purpose of the sensitivity analysis was to show the effect of errors in observed head, pumpage, transmissivity and storativity on the predicted head values calculated by the mathematical model developed herein.

2.4.2 Identification Model Calibration

The calibration of the model was done for the Fairfield-New Baltimore aquifer system. Spieker (1968) and Miami Conservancy District, Dayton, Ohio furnished the basic hydrogeological data for this system. The time period 1952 to 1962 was chosen for the identification and validation processes and was used in this way:

(1) 1952-1956 for model identification

(2) 1956-1962 for model validation

Observed water heads at different grid points of cells #4, #5 and #6 were generated for 1952 to 1956 using Spieker's mathematical model, parameters and conditions that he determined for the same problem area. This provided water head estimates for the six pumping wells of the region which were used for individual cell parameter identification of this work. Generated water head observations are presented in Tables 2.4(a), 2.4(b) and 2.4(c).

The identification algorithm was started using the initial guess of transmissivity averaged between cells as follows:

$$T_1 = 0.25$$

$$T_2 = 0.51$$

$$T_3 = 0.907$$

$$T_4 = 0.915$$

$$T_5 = 0.649$$

$$T_6 = 0.412$$

$$T_7 = 0.36$$

$$T_8 = 0.201$$

$$T_9 = 0.663$$

$$T_{10} = 0.66$$

$$T_{11} = 0.62$$

$$T_{12} = 0.209$$

Where subscripts 1,......12 of T mentioned above represent the following flow relation between cells (See Fig. 2.5)

Subscripts	Flow Relation Between Cells
1	2←1
2	2←4
3	4←3
4	4←10
5	6←5
6	5←7
7	7←6
8	7←8
9	6←9
10	5←10
11	5←11
12	7←12

The initial guess of transmissivity is based on the geological information of that area. The aquifer was simulated by the multicell model to produce:

(i) the interflow between adjacent cells

(ii) an averaged water level in all cells

For the five-year period (1952-1956) using initial guesses, parameters (b_1,b_2,b_3,b_4,b_5) of transmissivity function

$$T(x,y) = b_1x^2 + b_2y^2 + b_3x + b_4y + b_5$$

of cells #4, #5 and #6 were identified after being subjected to the above information developed in the iterative process.

Computationally, the identification scheme of this work is very effective. However, the initial guess of transmissivity plays a dominant role in computation time. The least-square error function between observed and calculated head of each cell converges quadratically to a minimum even with bad initial values (corresponding to a large initial least-square error). The model-predicted drawdowns for 1952 to 1956 are shown in Tables 2.5(a), 2.5(b) and 2.5(c). A comparison of the real (observed) drawdown values and the model's predicted drawdown (Tables 2.6(a), 2.6(b) and 2.6(c)) shows generally good agreement between them. Results of the identification of transmissivity function parameters are tabulated in Table 2.7.

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdown(FT)	Pumping Period=5(1956) Drawdowns(FT)
4 8	-0.801	-1.368	-1.511	-1.542	-1.251
5 7	-0.423	-0.801	-0.965	-1.001	-0.309
5 8	-1.201	-1.913	-2.141	-2.192	-1.586
6 7	-0.204	-0.572	-0.752	-0.801	0.392
7 5	1.056	1.092	1.033	1.016	2.721
7 6	3.273	3.231	3.124	3.092	5.770
8 4	0.722	0.795	0.778	0.761	1.864
8 5	3.541	3.662	3.612	3.599	6.301
8 6	3.839	3.915	3.837	3.805	7.351

Water Head Observations of Cell #4
(generated after Spieker)

Table: 2.4(a)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns (FT)
7 6	-0.989	-1.528	-1.643	-1.960	-1.979
7 7	-0.305	-0.521	-0.583	-0.705	-0.726
8 6	-1.218	-1.699	-0.142	-2.077	-2.045
9 4	-5.595	-7.247	-7.357	-8.558	-8.330
9 5	-11.385	-13.771	-13.516	-15.852	-15.233

Water Head Observations of Cell #5
(generated after Spieker)

TABLE: 2.4(b)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT
4 6	-0.198	-0.232	-0.248	-0.245	-0.245
5 6	-0.482	-0.547	-0.576	-0.571	-0.570
6 7	-4.944	-5.038	-5.087	-5.075	-5.072
6 8	-1.134	-1.174	-1.238	-1.214	-1.211
7 5	-0.237	-0.292	-0.320	-0.317	-0.316
7 7	-0.991	-1.075	-1.128	-1.116	-1.113
8 5	-0.126	-0.173	-0.201	-0.119	-0.197
8 7	-0.576	-0.647	-0.702	-0.690	-0.687

Water Head Observations of Cell #6
(generated after Spieker)

TABLE: 2.4(c)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT)
4 8	-0.791201	-1.418124	-1.821123	-1.552213	-1.461061
5 7	-0.442321	-0.861231	-1.115321	-1.021241	-0.339420
5 8	-1.351347	-2.031471	-2.561246	-2.302120	-1.629146
6 7	-0.413424	-0.552139	-0.962124	-0.841216	-0.512344
7 5	1.021236	1.132134	1.073659	1.166122	-2.741932
7 6	3.373432	3.241416	3.144630	3.292243	6.149243
8 4	1.019234	1.205618	0.808357	1.041642	2.124128
8 5	3.769213	3.922412	3.822124	3.629624	6.311426
8 6	4.091456	3.925243	4.097162	3.905271	7.501460

Cell #4 Water Head Predicted by the Model

TABLE: 2.5(a)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT)
7 6	-1.011012	-1.538213	-1.652134	-1.981245	-1.991234
7 7	-0.315112	-0.542641	-0.681235	-0.728634	-0.766198
8 6	-1.328431	-1.782145	-0.156143	-2.331240	-2.056231
9 4	-5.825120	-7.366123	-7.567916	-8.577421	-8.531041
9 5	-11.415341	-13.972034	-13.646450	-16.121456	-15.281468

Cell #5 Water Head Predicted by the Model

TABLE: 2.5(b)

Observation Point	Pumping Period=1(1952) Drawdowns(FT)	Pumping Period=2(1953) Drawdowns(FT)	Pumping Period=3(1954) Drawdowns(FT)	Pumping Period=4(1955) Drawdowns(FT)	Pumping Period=5(1956) Drawdowns(FT
4 6	-0.225143	-0.252164	-0.259942	-0.232114	-0.442143
5 6	-0.572261	-0.681432	-0.562143	-0.591241	-0.583264
6 7	-5.213462	-5.224126	-5.386432	-5.171242	-5.291348
6 8	-1.321420	-1.191264	-1.352684	-1.525146	-1.401342
7 5	-0.248168	-0.308148	-0.517941	-0.422136	-0.328116
7 7	-1.213480	-1.086142	-1.153121	-1.125334	-1.724321
8 5	-0.145321	-0.576452	-0.227418	-0.231468	-0.212346
8 7	-0.591242	-0.665432	-1.031402	-0.841531	-0.883451

Cell #6 Water Head Predicted by the Model

TABLE 2.5(c)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-0.801	-0.791201	0.01
5	7	-0.423	-0.442321	0.02
5	8	-1.201	-1.351347	0.15
6	7	-0.204	-0.413424	0.21
7	5	1.056	1.021236	0.03
7	6	3.273	3.373432	0.10
8	4	0.722	1.019234	0.30
8	5	3.541	3.769213	0.22
8	6	3.839	4.091456	0.26

Cell #4 Water Head Comparison

TABLE: 2.6(a)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.368	-1.418124	0.05
5	7	-0.801	-0.861231	0.06
5	8	-1.913	-2.031471	0.12
6	7	-0.572	-0.552139	0.02
7	5	1.092	1.132134	0.04
7	6	3.231	3.241416	0.01
8	4	0.795	1.205618	0.41
8	5	3.662	3.922412	0.26
8	6	3.915	3.925243	0.01

TABLE: 2.6(a)
(Continued)

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.511	-1.821123	0.31
5	7	-0.965	-1.115321	0.15
5	8	-2.141	-2.561246	0.42
6	7	-0.752	-0.962124	0.21
7	5	1.033	1.073659	0.04
7	6	3.124	3.144630	0.02
8	4	0.778	0.808357	0.03
8	5	3.612	3.822124	0.21
8	6	3.837	4.097162	0.26

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$ (FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.542	-1.552213	0.01
5	7	-1.001	-1.021241	0.02
5	8	-2.192	-2.302120	0.11
6	7	-0.801	-0.841216	0.04
7	5	1.016	1.166122	0.15
7	6	3.092	3.292243	0.20
8	4	0.761	1.041642	0.28
8	5	3.599	3.629624	0.03
8	6	3.805	3.905271	0.10

TABLE: 2.6(a)
(Continued)

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	8	-1.251	-1.461061	0.21
5	7	-0.309	-0.339420	0.03
5	8	-1.586	-1.629146	0.04
6	7	0.392	0.512344	0.12
7	5	2.721	-2.741932	0.02
7	6	5.770	6.149243	0.37
8		1.864	2.124128	0.26
8	5	6.301	6.311426	0.01
8	6	7.351	7.501460	0.15

TABLE: 2.6(a)
(Continued)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted by Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-0.989	-1.011012	0.02
7	7	-0.305	-0.315112	0.01
8	6	-1.218	-1.328431	0.11
9	4	-5.595	-5.825120	0.23
9	5	-11.385	-11.415341	0.03

Cell #5 Water Head Comparison

TABLE: 2.6(b)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.528	-1.538213	0.01
7	7	-0.521	-0.542641	0.02
8	6	-1.699	-1.782145	0.09
9	4	-7.247	-7.366123	0.12
9	5	-13.771	-13.972034	0.2

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.643	-1.652134	0.09
7	7	-0.583	-0.681235	0.1
8	6	0.142	-0.156143	0.01
9	4	-7.357	-7.567916	0.21
9	5	-13.516	-13.646450	0.13

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.960	-1.981245	0.02
7	7	-0.705	-0.728634	0.02
8	6	-2.077	-2.331240	0.26
9	4	-8.558	-8.577421	0.01
9	5	-15.852	-16.121456	0.27

TABLE: 2.6(b)
(Continued)

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
7	6	-1.979	-1.991234	0.02
7	7	-0.726	-0.766198	0.04
8	6	-2.045	-2.056231	0.01
9	4	-8.330	-8.531041	0.20
9	5	-15.233	-15.281468	0.05

TABLE: 2.6(b)
(Continued)

Pumping Period = 1(1952)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.198	-0.225143	0.02
5	6	-0.482	-0.572261	0.09
6	7	-4.944	-5.213462	0.30
6	8	-1.134	-1.321420	0.19
7	5	-0.237	-0.248168	0.01
7	7	-0.991	-1.213480	0.22
8	5	-0.126	-0.145321	0.01
8	7	-0.576	-0.591242	0.01

Cell #6 Water Head Comparison

TABLE: 2.6(c)

Pumping Period = 2(1953)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.232	-0.252164	0.02
5	6	-0.547	-0.681432	0.14
6	7	-5.038	-5.224126	0.19
6	8	-1.174	-1.191264	0.02
7	5	-0.292	-0.308148	0.01
7	7	-1.075	-1.086142	0.01
8	5	-0.173	-0.576452	0.40
8	7	-0.647	-0.6654432	0.02

Pumping Period = 3(1954)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.248	-0.259942	0.01
5	6	-0.576	-0.562143	0.01
6	7	-5.087	-5.386432	0.30
6	8	-1.238	-1.352684	0.12
7	5	-0.320	-0.517941	0.19
7	7	-1.128	-1.153121	0.03
8	5	-0.021	-0.227418	0.02
8	7	-0.702	-1.031402	0.33

TABLE: 2.6(c)
(Continued)

Pumping Period = 4(1955)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.245	-0.232114	0.01
5	6	-0.571	-0.591241	0.02
6	7	-5.075	-5.171242	0.10
6	8	-1.214	-1.525146	0.31
7	5	-0.317	-0.422136	0.11
7	7	-1.116	-1.125334	0.01
8	5	-0.119	-0.231468	0.04
8	7	-0.690	-0.841531	0.25

Pumping Period = 5(1956)

Observation Point		Water Head Predicted By Spiekers Model $\hat{h}$(FT.)	Water Head Predicted By Sarkars Model h(FT.)	$\Delta = h - \hat{h}$
4	6	-0.245	-0.442143	0.20
5	6	-0.570	-0.583264	0.01
6	7	-5.072	-5.291348	0.22
6	8	-1.211	-1.401342	0.19
7	5	-0.316	-0.328116	0.01
7	7	-1.113	-1.724321	0.61
8	5	-0.197	-0.212346	0.15
8	7	-0.687	-0.883451	0.20

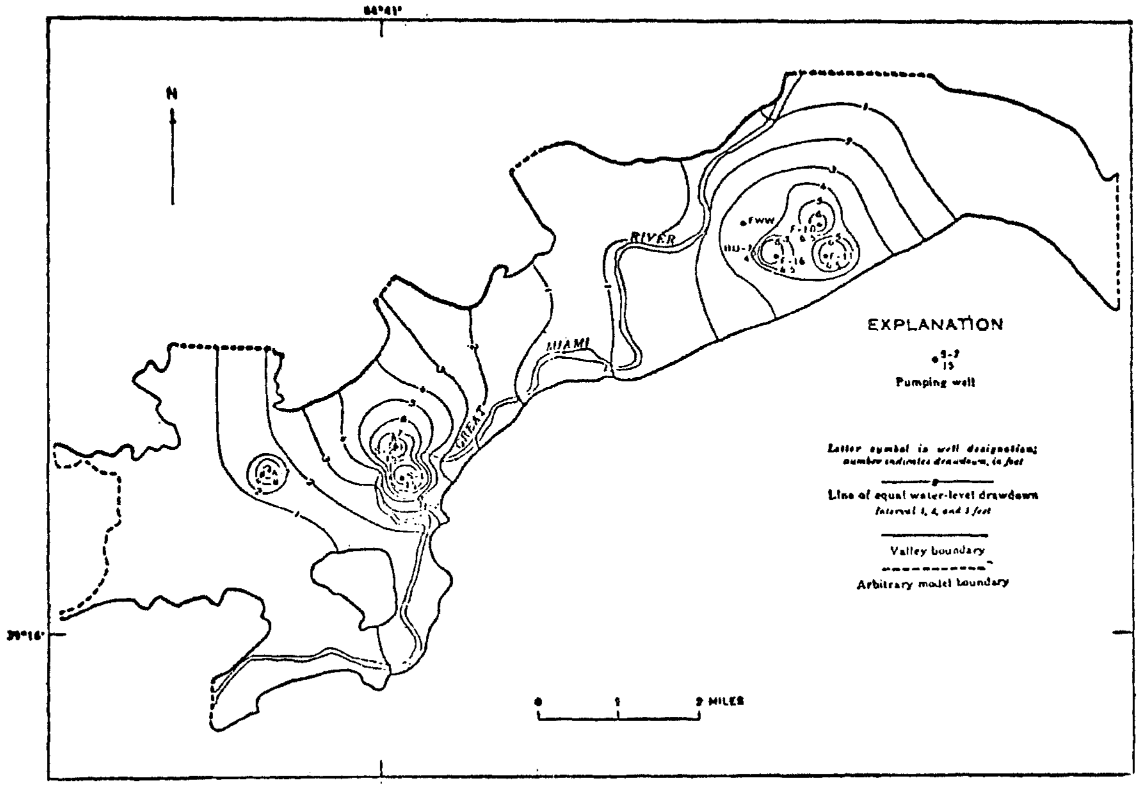
TABLE: 2.6(c)
(Continued)

ranges between 2% and 15%. However the predicted drawdown in Phases I and II varies from 15% to 33% and 5% to 31% respectively for those same well locations. This implies an impressive improvement in predictive ability was obtained in this work due to its decomposed modeling approach of using additional information to obtain an overall better model yielding more accurate results.

PARAMETERS	CELL #4	CELL #5	CELL #6
b_1	$.2132\times10^{-10}$	$.1245\times10^{-11}$	$-.4013\times10^{-11}$
b_2	$.1013\times10^{-11}$	$-.1300\times10^{-11}$	$.2132\times10^{-10}$
b_3	$.4121\times10^{-6}$	$.2140\times10^{-8}$	$.3012\times10^{-7}$
b_4	$.8234\times10^{-7}$	$.1611\times10^{-8}$	$.5034\times10^{-7}$
b_5	.6	.56	.46

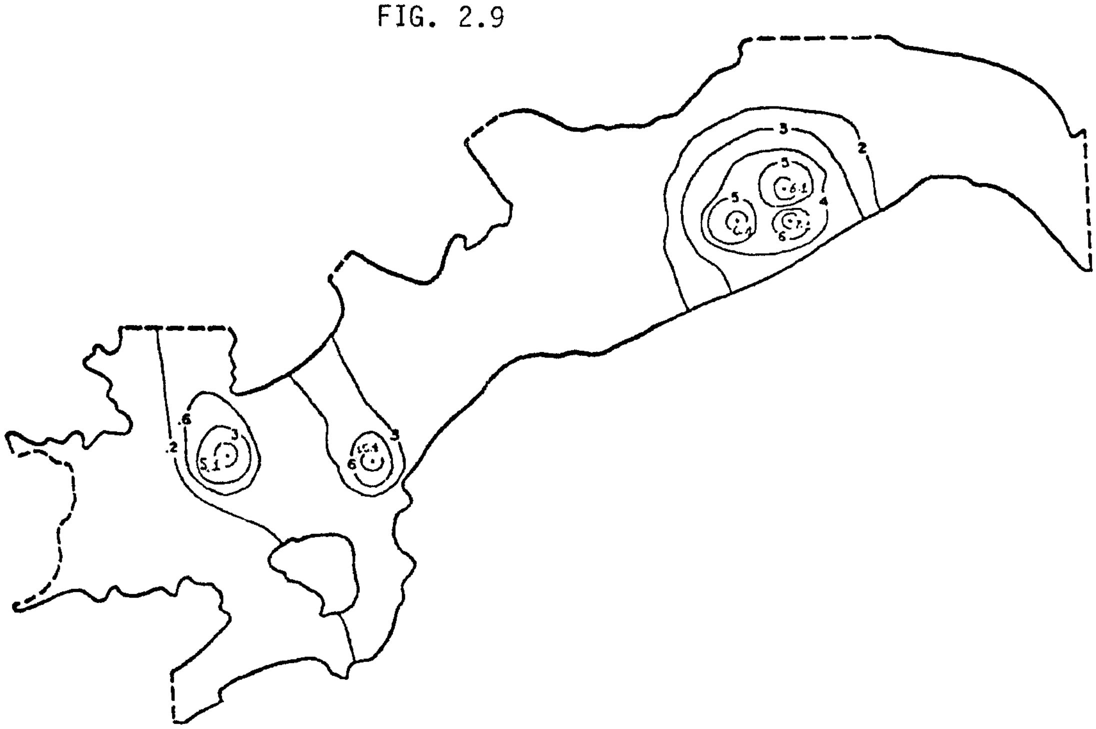
TABLE: 2.7

Results of the Identification of Cell #4, #5 & #6 of the Fairfield-New Baltimore Aquifer System



Drawdowns Caused by Pumping for the
Period 1952-62. Real System Observations made on
November 1962 (After Speieker)

FIG. 2.9



Drawdowns Caused by Pumping for The
Period 1952-62, Based on Decomposed Model
Derived in this Thesis

FIG. 2.10

Well Name	Cell Location	Observed Head(ft) After Spieker	Multicell Concept T Quadratic Phase III			Singlecell Concept T Quadratic Phase II			Singlecell Concept T Linear Phase I		
			Predicted Head(ft) h	Difference in $h \sim \hat{h}$	Error (%)	Predicted Head(ft) h	Difference in $h \sim \hat{h}$	Error (%)	Predicted Head(ft)	Difference in $h \sim \hat{h}$	Error (%)
A-2	6	6.0	5.09	0.1	15.0	4.15	1.85	31.0	4.0	2.0	33.0
S1-S2	5	15.0	15.4	0.45	3.0	12.0	3.0	20.0	12.0	3.0	20.0
F-16	4	6.5	6.49	0.01	2.0	6.14	0.36	5.6	7.7	1.2	18.4
F-10	4	6.5	6.08	0.42	6.0	6.05	0.45	6.93	7.5	1.0	15.3
F-11	4	6.5	7.08	0.58	10.0	7.40	0.9	14.0	8.7	2.2	30.0

TABLE: 2.8

Results of the Fairfield-New Baltimore Aquifer

Model Forecasted Results

(Water Heads Compared on November 1962)

2.4.4 Sensitivity Analysis

2.4.4.1 *Introduction*

Generally hydrologic phenomena are affected by complex natural events, the details of which cannot be anticipated precisely. Hence the analysis of hydrologic systems is often viewed in terms of stochastic processes. However, the analysis of groundwater flow has traditionally been based on a deterministic approach to the solution of the governing partial differential equation. Natural variability, such as temporal fluctuations in groundwater recharge, storativity, infiltration, evapotranspiration and spatial variation in transmissivity, is usually dealt with only in terms of average conditions. Yet natural variability may be an important feature of groundwater flow in that it may be possible to infer aquifer properties from water table fluctuations.

In the following analysis, effect of temporal variability in various groundwater system parameters on hydraulic head values of the Fairfield-New Baltimore aquifer are examined. Before the development of different optimization methodologies used for ground water parameter identification, this type of analysis was also used for precise estimation of these parameters. In this work, various sensitivity analyses were performed to determine the effect of errors in transmissivity, storativity, observed head and pumpage on model prediction. The resulting sensitivity and statistical analyses as discussed in the following section were found to be useful in finding which parameter must be specified with the greatest accuracy in order to model adequately the groundwater system, and which parameter of the groundwater system is causing most sensitivity on the model water head prediction.

2.4.4.2 *Effect of Errors in Storativity on Model Water Head Prediction*

A sensitivity analysis was performed to determine the effect of error in storativity on the parameter values and its influence on waterhead prediction. The behavior of model waterhead prediction at five well locations due to the small change in storage coefficients of different cells (Cells #4, #5 and #6) was studied. For the bulk of the area covered by Cells #4 and #5, where the groundwater occurs under unconfined conditions, the storativity was perturbed around a value of 0.2 ($S_1 = 0.15$, $S_2 = 0.2$, $S_3 = 0.25$) which is a typical value for an unconfined aquifer. In the area covered by Cell #6, the storativity was perturbed around 0.1 ($S_1 = 0.07$, $S_2 = 0.1$, $S_3 = 0.15$), because here, although the groundwater is largely unconfined, a thin layer of clay locally separates the aquifer into two parts (Spieker, 1968). This separation is considered to reduce the storativity to slightly less than the normal value of 0.2 associated with unconfined conditions.

Table 2.9(a)-2.9(c) shows the sensitivity analyses for five well locations. This required three solutions of the identification algorithm and three corresponding solutions for computing waterhead prediction. A statistical analysis of error in waterhead prediction due to change in storativity was also performed (See Table 2.9(d)). The analysis indicated that under a varying range of error in storativity ($\pm$ 25% of average value), the percentage error in waterhead prediction has mean value (μ) in the range of 0 to -12 and standard deviation (σ) 0 to 0.01. This shows that in general the deviation of output at different well locations is not appreciably sensitive to the change in the storativity parameter. It has also been noted that in two well locations (S1-S2 and A-2) the % of error in waterhead prediction is zero even where the percentage of error in storativity lies in the range of -30% to +30%. The conclusion of less sentitive output due to change in storativity holds equally for constant and varying pumping conditions. However the error in predicting output depends not only on storativity exclusively but also on other hydrologic phenomena in an aquifer.

2.4.4.3 *Effect of Errors in Observed Drawdown on Model Waterhead Prediction*

To evaluate the effect on model prediction due to the errors in observed drawdown, a sensitivity analysis was also performed. The identification problem was rerun with error artificially introduced in drawdown at five pumping well locations (F-10, F-11, F-16, S1-S2 and A-2). Table 2.10(a)-2.10(c) demonstrates results of this analysis. H_2 represents the computed head values when no error was introduced in the observed head under optimal conditions, whereas H_1 and H_3 represent the computed head values when different sets of error were introduced into the observed head. It was noted according to a statistical analysis (see Table 2.10(d)) that under various percentages of error ($\pm$ 5%) in observed head, the mean (μ) and standard deviation (σ) of percentage error in waterhead prediction varies from 12 to -14 and 0 - 0.11, respectively. This reveals that computed head values are moderately sensitive to error in observed drawdown. Generally more error in observed head results in more inaccurate waterhead forecasting. Although the results for only two sets of error are shown in Table 2.10(d), many other sets of error were examined and no exceptions to the aforementioned conclusions were found.

2.4.4.4 *Effect of Errors in Pumpage on Waterhead Prediction*

A sensitivity analysis was also performed to evaluate the effect on the parameter values identified and model prediction due to the error in pumpage at different wells in the aquifer. This is especially important since in a water resource system the rate of pumping varies for different reasons. The identification problem was also rerun with changed pumping. This yielded the effect of this change on the optimal parameter values causing different waterhead predictions (See Table 2.11(a)-2.11(c)). A statistical analysis of errors in pumpage (See Table 2.11(d)) indicates that under its various percentage error ($\pm$ 10%), the mean (μ) and standard deviation (σ) of percentage error in waterhead prediction varies in a range of 8 to -17 and 0.02 to 0.08, respectively. The results of this analysis also demonstrate that the computed head values are closely related to the amount of pumpage error. Generally more error in pumpage will result in more drawdown and vice versa. However this relationship does not follow any particular pattern due to the various geological characteristics of the aquifer which affect waterhead drawdown.

2.4.4.5 *Effect of Errors in Transmissivity on Waterhead Prediction*

As mentioned earlier, transmissivity is an important property in a groundwater system. Its accurate estimation plays a dominant part in forecasting groundwater system response to various hydrologic stresses. To evaluate the effect of inaccurate estimation of the transmissivity parameter on waterhead prediction, a sensitivity analysis was done. This analysis was carried out by changing parameters representing transmissivity function T(x,y). As mentioned earlier transmissivity is approximated by a second-order polynomial function

$$T(x,y) = b_1x^2 + b_2y^2 + b_3x + b_4y + b_5$$

since it is known that the parameter b_5 of above equation has more weight in the function than any other parameters, e.g., b_1, b_2, b_3 & b_4.

Hence this parameter (b_5) was slightly changed around its optimal value, keeping other optimal parameters constant. The behavior of model waterhead prediction at five well locations due to this small change in transmissivity coefficient parameters was studied by means of statistical analysis (See Table 2.12(d)). The analysis indicated that under a range of error in transmissivity ($\pm$ 9% of its optimal value), the percentage error in waterhead prediction has mean value (μ) and standard deviation (σ) in the range of 16 to -17 and 0 to 0.03, respectively. This shows that in general: (i) the model waterhead prediction is quite sensitive to change in transmissivity and (ii) as transmissivity increases, the waterhead drawdown tends to decrease and vice versa. This is particularly true within the semiconfined aquifer zone (Well A-2) which is similar to the characteristics shown for the unconfined aquifer zone (Well F-10, F-11, F-16 and S1-S2) of the Fairfield-New Baltimore area.

2.4.4.6 *Comparative Statistical Analysis of Errors*

On the basis of the results of the statistical analyses just examined, a comparative study of the effect of errors in different parameters on waterhead prediction was made by answering the following problem. Let ε_h, ε_s, ε_{oh}, ε_p and ε_T be the percentage error of waterhead response (drawdown), storativity, observed head, pumpage and transmissivity respectively. Show how much ε_h varies for certain values of ε_s, ε_{oh}, ε_p and ε_T

Define

$E(\varepsilon_h/\varepsilon_s)$ = Expected value of error in response to given error in storativity.

$E(\varepsilon_h/\varepsilon_{oh})$ = Expected value of error in response to given error in observed head.

$E(\varepsilon_h/\varepsilon_p)$ = Expected value of error in response to given error in pumpage.

$E(\varepsilon_h/\varepsilon_T)$ = Expected value of error in response to given error in transmissivity.

Considering Well (A-2) for the present study and collecting information from Table 2.9(d), 2.10(d), 2.11(d) and 2.12(d) we have

$$E(\varepsilon_h/\varepsilon_s = 30) = 0$$

$$E(\varepsilon_h/\varepsilon_{oh} = 9) = 1.0$$

$$E(\varepsilon_h/\varepsilon_p = 10) = 4.0$$

$$E(\varepsilon_h/\varepsilon_T = 9) = 17.0$$

The above statistical statement clearly explains that in the present case 9% of the error in transmissivity has 17% of the error in response while

(i) 30% of error in storativity has no error in response

(ii) 9% of error in observed head has 1% of error in response

(iii) 10% of error in pumpage has 4% of error in response.

Thus above sensitivity and statistical analyses establish the following facts:

(1) In general the modeling technique of this chapter is less sensitive to change in parameters.

(2) Waterhead prediction is more sensitive to change in transmissivity than to change in any other parameters. Hence if transmissivity of a model is not quite accurately known, the model output becomes erroneous.

WELL NAME	STORATIVITY		YEAR	DRAWDOWNS (FT)
F-10	S_1	0.15	1952	2.98
			1953	2.89
			1954	2.79
			1955	2.77
			1956	5.33
	S_2	0.2	1952	3.26
			1953	3.22
			1954	3.11
			1955	3.08
			1956	5.76
	S_3	0.25	1952	3.58
			1953	3.57
			1954	3.39
			1955	3.41
			1956	6.40
F-11	S_1	0.15	1952	3.51
			1953	3.52
			1954	3.44
			1955	3.41
			1956	6.78
	S_2	0.2	1952	3.82
			1953	3.90
			1954	3.82
			1955	3.79
			1956	7.34
	S_3	0.25	1952	4.20
			1953	4.29
			1954	4.16
			1955	4.21
			1956	8.14
F-16	S_1	0.15	1952	3.24
			1953	3.29
			1954	3.24
			1955	3.22
			1956	5.80
	S_2	0.2	1952	3.53
			1953	3.65
			1954	3.60
			1955	3.58
			1956	6.29
	S_3	0.25	1952	3.99
			1953	4.09
			1954	4.03
			1955	4.04
			1956	7.10

TABLE: 2.9(a)

CELL #4

Results of Sensitivity Analysis

Effect of Errors in Storativity on Water Head Prediction

WELL NAME	STORATIVITY		YEAR	DRAWDOWN (FT)
S1-S2	S_1	.15	1952	11.55
			1953	13.86
			1954	13.57
			1955	15.92
			1956	15.27
	S_2	.2	1952	11.41
			1953	13.80
			1954	13.56
			1955	15.89
			1956	15.27
	S_3	.25	1952	11.52
			1953	13.85
			1954	13.58
			1955	15.92
			1956	15.27

TABLE: 2.9(b)

CELL #5

Effect of Errors in Storativity on Water Head Prediction

WELL NAME	STORATIVITY		YEAR	DRAWDOWN (FT)
A-2	S_1	.07	1952	4.99
			1953	5.01
			1954	5.07
			1955	5.07
			1956	5.07
	S_2	.1	1952	4.94
			1953	5.03
			1954	5.08
			1955	5.07
			1956	5.07
	S_3	.15	1952	5.68
			1953	5.04
			1954	5.09
			1955	5.07
			1956	5.07

TABLE: 2.9(c)

CELL #6

Effect of Errors in Storativity on Waterhead Prediction

Well	% Error of Storativity	Year	% Error	Drawdowns % Error Mean (μ)	Standard Deviation (σ)	Variance (σ^2)
F-10	-25	1952	- 9.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	11.0	0.01	δ
		1953	11.0			
		1954	9.0			
		1955	11.0			
		1956	11.0			
F-11	-25	1952	- 8.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	10.0	0.01	δ
		1953	10.0			
		1954	9.0			
		1955	11.0			
		1956	11.0			
F-16	-25	1952	- 8.0	-9.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 8.0			
	25	1952	10.0	12.0	0.01	δ
		1953	12.0			
		1954	12.0			
		1955	13.0			
		1956	13.0			
S1-S2	-25	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
	25	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
A-2	-30%	1952	1.0	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			
	30%	1952	1.5	δ	δ	δ
		1953	0			
		1954	0			
		1955	0			
		1956	0			

$\delta < 0.001$

Statistical Analysis of Errors in Storativity

TABLE: 2.9(d)

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
At Well Location F-11 -7% of 1952 +5% of 1953 +12% of 1954 -9% of 1955 +4% of 1956	H_1	F-10	3.5	3.0	2.86	3.48	5.11
		F-16	3.38	3.25	3.73	3.84	5.77
		F-11	3.5	3.14	3.31	3.0	6.10
No Error	H_2	F-10	3.26	3.22	3.11	3.08	5.76
		F-16	3.53	3.65	3.60	3.58	6.10
		F-11	3.62	3.90	3.82	3.65	7.16
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	F-10	3.22	3.55	3.2	4.0	4.7
		F-16	3.58	3.77	3.92	3.75	5.5
		F-11	4.25	4.49	4.0	4.24	8.01

TABLE 2.10(a)

Cell #4

Sensitivity Analysis

Effect of Errors in Observed Drawdown On Water Head Prediction

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
At Well Location S1-S-2 -7% of 1952 5% of 1953 12% of 1954 -9% of 1955 -4% of 1956	H_1	S1-S2	12.0	14.42	13.25	16.4	15.52
No Error	H_2	S1-S2	11.41	13.80	13.56	15.89	15.27
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	S1-S2	11.65	13.4	13.7	15.57	14.8

TABLE: 2.10(b)

Cell #5

Sensitivity Analysis

Effect of Errors in Waterhead Observation on Waterhead Prediction

Percentage Of Error Introduced In Water Head Observation		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
At Well Location A-2 -7% of 1952 5% of 1953 12% of 1954 -9% of 1955 +4% of 1956	H_1	A-2	4.98	5.07	5.12	5.11	5.04
No Error	H_2	A-2	4.94	5.03	5.08	5.07	5.07
7% of 1952 -5% of 1953 -12% of 1954 9% of 1955 -4% of 1956	H_3	A-2	4.88	4.97	5.06	5.13	5.11

TABLE: 2.10(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Waterhead Observation on Waterhead Prediction

Well	% Error of Observed Head	Year	% Error	Drawdowns % Error Mean (μ)	Standard Deviation (σ)	Variance (σ^2)
F-10	5%	1952	7.0	-1.0	0.11	0.012
		1953	-7.0			
		1954	-8.0			
		1955	13.0			
		1956	-11.0			
	-5%	1952	-7.0	5.0	0.12	0.014
		1953	10.0			
		1954	8.0			
		1955	14.0			
		1956	-6.0			
F-11	5%	1952	-5.0	-4.0	0.02	δ
		1953	-5.0			
		1954	-4.0			
		1955	-4.0			
		1956	-4.0			
	-5%	1952	2.0	2.0	0.04	δ
		1953	3.0			
		1954	2.0			
		1955	2.0			
		1956	3.0			
F-16	5%	1952	-4.0	-2.0	0.07	δ
		1953	-11.0			
		1954	4.0			
		1955	7.0			
		1956	-5.0			
	-5%	1952	1.0	2.0	0.07	δ
		1953	3.0			
		1954	9.0			
		1955	5.0			
		1956	-10.0			
S1-S2	5%	1952	5.0	2.0	0.03	δ
		1953	4.0			
		1954	-2.0			
		1955	3.0			
		1956	2.0			
	-5%	1952	2.0	-1.0	0.02	δ
		1953	-3.0			
		1954	1.0			
		1955	-2.0			
		1956	-3.0			
A-2	9%	1952	1.0	1.0	0.01	δ
		1953	1.0			
		1954	1.0			
		1955	1.0			
		1956	-1.0			
	-9%	1952	-1.0	δ	δ	δ
		1953	-1.0			
		1954	0			
		1955	1.0			
		1955	1.0			

$\delta < 0.001$

Statistical Analysis of Errors in Observed Head

TABLE 2.10(d)

Percentage Of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	F-10	2.68	2.72	2.42	2.45	4.84
		F-16	2.98	3.17	2.96	3.02	5,33
		F-11	3.14	3.33	3.0	3.05	6.19
No Error	P_2	F-10	3.26	3.22	3.11	3.08	5.76
		F-16	3.53	3.65	3.60	3.58	6.10
		F-11	3.82	3.90	3.82	3.65	7.16
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	F-10	3.39	3.44	3.12	3.18	5.88
		F-16	3.67	3.86	3.88	4.11	7.37
		F-11	3.89	3.92	3.85	3.66	7.37

TABLE: 2.11(a)

Cell #4

Percentage of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	S1-S2	10.51	13.21	11.95	14.52	14.23
No Error	P_2	S1-S2	11.41	13.80	13.56	15.89	15.27
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	S1-S2	12.31	14.4	15.14	17.25	16.31

TABLE: 2.11(b)

Cell #5

Percentage of Error Introduced In Pumping		Well Name	Drawdowns (FT) 1952	1953	1954	1955	1956
-10% of 1952 -5% of 1953 -15% of 1954 -10% of 1955 -8% of 1956	P_1	A-2	4.76	4.92	4.8	4.87	4.76
No Error	P_2	A-2	4.94	5.03	5.08	5.07	5.07
10% of 1952 5% of 1953 15% of 1954 10% of 1955 8% of 1956	P_3	A-2	5.12	5.14	5.36	5.27	5.37

TABLE: 2.11(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Pumpage on Waterhead Prediction

Well	% Error of Pumpage	Year	Drawdowns % Error			
			% Error	Mean (μ)	Standard Deviation (σ)	Variance (σ^2)
F-10	-10%	1952	-18.0	-18.0	0.03	δ
		1953	-16.0			
		1954	-22.0			
		1955	-20.0			
		1956	-16.0			
	10%	1952	4.0	3.0	0.03	δ
		1953	7.0			
		1954	0			
		1955	0			
		1956	0			
F-11	-10%	1952	-18.0	-17.0	0.03	δ
		1953	-15.0			
		1954	-21.0			
		1955	-16.0			
		1956	-14.0			
	10%	1952	4.0	6.0	0.07	δ
		1953	6.0			
		1954	8.0			
		1955	15.0			
		1956	3.0			
F-16	-10%	1952	-16.0	-15.0	0.02	δ
		1953	-13.0			
		1954	-18.0			
		1955	-16.0			
		1956	-13.0			
	10%	1952	2.0	1.0	0.03	δ
		1953	5.0			
		1954	1.0			
		1955	0			
		1956	3.0			
S1-S2	-10%	1952	- 8.0	- 4.0	0.08	0.01
		1953	- 4.0			
		1954	-12.0			
		1955	- 9.0			
		1956	- 7.0			
	10%	1952	8.0	8.0	0.03	δ
		1953	4.0			
		1954	12.0			
		1955	8.0			
		1956	7.0			
A-2	-10%	1952	- 4.0	- 4.0	0.02	δ
		1953	- 2.0			
		1954	- 6.0			
		1955	- 4.0			
		1956	- 6.0			
	10%	1952	4.0	4.0	0.02	δ
		1953	2.0			
		1954	6.0			
		1955	4.0			
		1956	6.0			

$\delta < 0.001$

Statistical Analysis of Errors in Pumpage

TABLE: 2.11(d)

Well Name		Transmissivity Parameters	Year	Drawdown (FT)
F-10	$T_1 = 0.58$	$b_1 = .2132X10^{-10}$	1952	3.67
		$b_2 = 1031X10^{-11}$	1953	3.64
		$b_3 = .4121X10^{-6}$	1954	3.52
		$b_4 = .8234X10^{-7}$	1955	3.48
		$b_5 = .57$	1956	6.44
	$T_2 = 0.61$	$b_1 = .2132X10^{-10}$	1952	3.26
		$b_2 = 1031X10^{-11}$	1953	3.22
		$b_3 = .4121X10^{-6}$	1954	3.11
		$b_4 = .8234X10^{-7}$	1955	3.08
		$b_5 = .6$	1956	5.76
	$T_3 = 0.64$	$b_1 = .2132X10^{-10}$	1952	2.96
		$b_2 = .1031X10^{-11}$	1953	2.90
		$b_3 = .4121X10^{-6}$	1954	2.80
		$b_4 = .8234X10^{-7}$	1955	2.78
		$b_5 = 63$	1956	5.23
F-11	$T_1 = 0.58$	$b_1 = .2132X10^{-10}$	1952	4.29
		$b_2 = .1031X10^{-11}$	1953	4.41
		$b_3 = .4121X10^{-6}$	1954	4.32
		$b_4 = .8234X10^{-7}$	1955	4.28
		$b_5 = .57$	1956	8.20
	$T_2 = 0.61$	$b_1 = .2132X10^{-10}$	1952	3.82
		$b_2 = 1031X10^{-11}$	1953	3.90
		$b_3 = .4121X10^{-6}$	1954	3.82
		$b_4 = .8234X10^{-7}$	1955	3.79
		$b_5 = .6$	1956	7.34
	$T_3 = 0.64$	$b_1 = .2132X10^{-10}$	1952	3.46
		$b_2 = .1031X10^{-11}$	1953	3.52
		$b_3 = .4121X10^{-6}$	1954	3.45
		$b_4 = .8234X10^{-7}$	1955	3.42
		$b_5 = .63$	1956	6.66
F-16	$T_1 = 0.58$	$b_1 = .2132X10^{-10}$	1952	3.96
		$b_2 = .1031X10^{-11}$	1953	4.11
		$b_3 = .4121X10^{-6}$	1954	4.07
		$b_4 = .8234X10^{-7}$	1955	4.04
		$b_5 = .57$	1956	7.04
	$T_2 = 0.61$	$b_1 = .2132X10^{-10}$	1952	3.53
		$b_2 = 1031X10^{-11}$	1953	3.65
		$b_3 = .4121X10^{-6}$	1954	3.60
		$b_4 = .8234X10^{-7}$	1955	3.58
		$b_5 = .6$	1956	6.29
	$T_3 = 0.64$	$b_1 = .2132X10^{-10}$	1952	3.20
		$b_2 = .1031X10^{-11}$	1953	3.29
		$b_3 = .4121X10^{-6}$	1954	3.25
		$b_4 = .8234X10^{-7}$	1955	3.23
		$b_5 = .63$	1956	5.25

TABLE: 2.12(a)
Cell #4
Sensitivity Analysis
Effect of Errors in Transmissivity on Waterhead Prediction

Well Name		Transmissivity	Year	Drawdown (FT)
S1-S2	T_1 = 0.53	b_1 = $1245X10^{-11}$	1952	13.21
		b_2 = $.1300X10^{-4}$	1953	16.06
		b_3 = $.2140X10^{-8}$	1954	15.81
		b_4 = $1611X10^{-8}$	1955	18.51
		b_5 = .53	1956	17.82
	T_2 = 0.56	b_1 = $1245X10^{-11}$	1952	11.41
		b_2 = $.1300X10^{-11}$	1953	13.80
		b_3 = $.2140X10^{-8}$	1954	13.56
		b_4 = $.1611X10^{-8}$	1955	15.89
		b_5 = .56	1956	15.27
	T_3 = 0.6	b_1 = $.1245X10^{-11}$	1952	10.5
		b_2 = $.1300X10^{-11}$	1953	13.0
		b_3 = $.2140X10^{-8}$	1954	12.5
		b_4 = $.1611X10^{-8}$	1955	14.0
		b_5 = .6	1956	13.4

TABLE: 2.12(b)

Cell #5

Well Name		Transmissivity Parameters	Year	Drawdown (FT)
A-2	T_1 = 0.42	b_1 =$-.4013X10^{-11}$	1952	5.51
		b_2 = $.2132X10^{-10}$	1953	5.55
		b_3 = $.3012X10^{-7}$	1954	5.58
		b_4 = $.5034X10^{-7}$	1955	5.56
		b_5 = .42	1956	5.52
	T_2 = 0.46	b_1 =$-.4013X10^{-11}$	1952	4.94
		b_2 = $.2132X10^{-10}$	1953	5.03
		b_3 = $.3012X10^{-7}$	1954	5.08
		b_4 = $.5034X10^{-7}$	1955	5.07
		b_5 = .46	1956	5.07
	T_3 = 0.50	b_1 =$-.4013X10^{-11}$	1952	4.14
		b_2 = $.2132X10^{-10}$	1953	4.20
		b_3 = $.3012X10^{-7}$	1954	4.24
		b_4 = $.5034X10^{-7}$	1955	4.22
		b_5 = .5	1956	4.22

TABLE: 2.12(c)

Cell #6

Sensitivity Analysis

Effect of Errors in Transmissivity on Waterhead Prediction

Well	% Error of Transmissivity	Year	% Error	Drawdowns % Error Mean (μ)	Standard Deviation (σ)	Variance (σ²)
F-10	-5	1952	13.0	13.0	δ	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	- 9.0	-10.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	- 9.0			
F-11	-5	1952	12.0	13.0	0.01	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	-10.0	-10.0	0.01	δ
		1953	-10.0			
		1954	-10.0			
		1955	-11.0			
		1956	- 9.0			
F-16	-5	1952	12.0	13.0	0.01	δ
		1953	13.0			
		1954	13.0			
		1955	13.0			
		1956	12.0			
	5	1952	- 9.0	-11.0	0.03	δ
		1953	-10.0			
		1954	-10.0			
		1955	-10.0			
		1956	-17			
S1-S2	-5	1952	11.0	10.0	0.01	δ
		1953	11.0			
		1954	10.0			
		1955	10.0			
		1956	10.0			
	5	1952	- 9.0	- 9.0	0.03	δ
		1953	- 6.0			
		1954	- 8.0			
		1955	-12.0			
		1956	-12.0			
A-2	-9	1952	17.0	15.0	0.01	δ
		1953	16.0			
		1954	15.0			
		1955	15.0			
		1956	15.0			
	9	1952	-16	-17.0	δ	δ
		1953	-17			
		1954	-17			
		1955	-17			
		1956	-17			

$\delta < 0.001$

Statistical Analysis of Errors in Transmissivity

TABLE: 2.12(d)

2.5 SUMMARY AND CONCLUSIONS

In recent works (Lopez, 1973; Lopez, Haimes and Das, 1974) represented in Phases I and II the parameter identification methodology of groundwater systems is essentially based on the observed input data and the associated response. However these methodologies do not use various existing information from the geological map of the system. This consequently leads to: (i) developing a mathematical model which becomes nonrepresentative of the real physical system and (ii) a slight change from the data base for such a system which results in a substantial fluctuation in model response.

The groundwater model variable for which various existing information is available includes (in addition to transmissivity) storativity, initial water levels, discharge, recharge, boundary conditions and topology. The model developed in this work utilizes the existing information so that the mathematical model is closely representative of the physical system. Its sensitivity to changes in the data is less compared to other models. The model was applied to a real groundwater system in southern Ohio. A systematic way of identifying the transmissivity function was developed by decomposing the system into blocks. This provides the systems analyst with the possibility of making use of the various hydrological information for identifying a parameter of different blocks. Besides being computationally superior to the methods developed by previous authors, this identification and model validation closely approximates the physical system (see Table 2.8). Approximation of the transmissivity function by a second-order polynomial function for each block provides a closer distribution of transmissivity values, since the transmissivity within a cell is somewhat homogeneous. The dynamic nature of the boundary conditions for each cell is more realistic in the modeling of groundwater systems. An error introduced, if due to gross approximation of boundary conditions, is not likely to be present in a multicell model.

Since a mass balance is seen for each cell in each time period, an error introduced by numerical approximation is confined to the system and thereby distributed in model output over the aquifer. This has also been observed by comparing the result of this phase with that of previous phases using the same data base and is shown in Table 2.8.

Identifying groundwater parameters of each cell involves solving a partial differential equation describing the flow in porous media by numerical approximation. Since the area of each cell is comparatively small, it provides us with finer grid points over each cell without increasing computational difficulty. This is because each cell model may be solved independent of the others. Hence the methodology developed in this work becomes computationally more tractable. The finer the grids the more accurate the numerical solutions.

Under the rather simplified decomposition approach of this chapter, the method developed for identifying transmissivity parameters from observed head values proved very accurate. However the accuracy of the results will be affected to a considerable degree by the choice of well locations within a cell at which the waterhead is observed.

The procedure developed for evaluating transmissivity was tested for as many as three cells. There is no apparent reason why the method could not be extended to a greater number of cells. It must be realized, however, that as the number of cells increases, the computer time and analysis time increases. The computer time for the identification algorithm of this chapter also depends on the guess of the average value of transmissivity parameters. Should the optimization process fail to produce a solution, the user will have to supply a new starting point. The information generated in unsuccessful runs can be used to make better initial guesses.

Concerning the core requirement, the program requires about 72K words on the Univac 1108 digital computer. As for computer time, with three cells (see Section 2.5, the Fairfield-New Baltimore Aquifer) and a period of five years (with yearly changes in pumpage rate) the program takes 112 seconds.

Sensitivity and statistical analyses applied to the case study reveal that the model is quite sensitive to changes in identified parameter (transmissivity) while less sensitive to other parameters (storativity, pumpage and observed head). Therefore it was decided that the only parameter to be identified would be transmissivity, which also compensates for errors in identifying other parameters.

CHAPTER 3

AN OPTIMAL CONTROL ANALYSIS FOR THE MANAGEMENT OF A GROUNDWATER AQUIFER-STREAM SYSTEM

3.1 A GENERAL DISCUSSION

The developments introduced in Phase II, Haimes [1974], and Chapter 1 of this study provide the basis for coupling a complex real physical system with any desired control scheme. The system may comprise both aquifers and a stream network, interacting throughout the basin. The control scheme may consider utilizing certain parts of or the entire water resource at the considered area. It may refer to an isolated subsystem, or to an administrative framework which is imposed on the regional structure. The main idea is that a controlled input such as pumpage or artificial recharge is subject to a decision process for its magnitude and distribution. This same input affects the physical system, which responds accordingly. The system response is directly and indirectly considered in the decision process, and hence embedded in this process is the feedback to the input from the system response to the output. Using the response functions in the form developed in Phase II allows for explicitly coupling the physical system response with the decision process. The functions are essentially the acting analytical tool whereby system response and controlled input are interrelated. It is therefore possible to construct a management model in which the input stress imposed is considered as a control variable. This variable is specified by the solution of the optimal control problem in the decision process.

In the following we intend to examine the management control problem formulation and the solution which should be applied to a system comprising a complex water resources system. In particular, we expect to demonstrate the real advantage of the response functions hierarchy while applied to mathematical models of the conjunctive use of ground and surface water systems.

This analysis is not available in the literature and constitutes a major contribution of this study. At first, an optimal control problem is formulated. The analysis of this problem should serve in better understanding the management model.

The effectiveness of using an optimal control theory for solving management models is well illustrated by Hullett, [1974], (for applying distributed parameter control theory to optimal estuary aeration). Unfortunately, the distributed parameter control system which is identified for the conjunctive use of ground and surface water is too complicated for successfully using existing optimal control theory, and hence some simplifications must be made. Analyzing the simplified problem provides some insight into certain features of the original problem, and evaluates some of the necessary conditions for an optimal solution. A numerical solution is proposed. It results from discretizing the distributed parameter control formulation of the mathematical model. Finally, in this chapter, a quadratic program resulting from applying the numerical analysis is discussed. The next chapter is devoted to the application of the mathematical model to the case study which has been analyzed throughout. Not all the features characterizing the management control model are identified in the case study area. However, to be close to reality, no additional generated information is assumed which would make the case more general. The application is restricted to the existing structure, reducing the model to a forecasting tool for future operations. It is found however to be of great interest by itself. Case 2 is then formulated. This is a hypothetical system featuring most of what is characterized by the management control mode. This case is aimed at illustrating the prospects of using that model for a full-scale conjunctive use of ground and surface water systems. Management models of great variety have been applied and used for optimal control in water resources systems. The response functions which are developed in our study should be applied in particular to a short-term planning model. Evidently, the functional relation between inputs such as pumping or recharge, and responses such as drawdown and interflow should mostly affect the operational aspects of the water resources development. The planning for capacity expansion is affected only through the aggregation of the operational effects. Models devoted to the capacity expansion problem are well developed. The coupling of the operational aspects as considered in the forthcoming model with a desired capacity expansion model is a straightforward task; however, this problem is beyond this study's scope. Buras, [1963], developed a dynamic programming algorithm to solve the problem of conjunctive use of reservoirs and aquifers. The operating policy considers the physical system in a lump form which introduces a considerable error by neglecting the distributed parameter system characterizing the groundwater system. As opposed to the lumped parameter approach, an analysis is suggested (Yu and Haimes, [1974]) whereby a multilevel formulation is used for explicitly coupling the distributed parameter system with a management scheme to optimize conjunctive use of ground and surface water. Maddock and Haimes, [1975], use the algebraic technological functions for coupling a groundwater system with a tax-quota management scheme. In the development below, conjunctive use of an aquifer system and a surface water system is considered. At this stage the regional administrative considerations will be included as well. However, regardless of the administrative structure, individual activities such as pumping from wells or consuming water from some common pools (like surface reservoirs), necessitate an information flow between people. Subject to such information, the single user is provided with the tools to make his own water use plans more efficiently and still maintain an independent operation policy.

3.2 THE REGIONAL SYSTEM

A basin comprising aquifers traversed by streams is considered. Users throughout the basin pump water from aquifers by means of operating wells. Each user's desire for water is primarily governed by economics, but may also take into consideration the stream water response, e.g., water level and quality in his vicinity. Surface water may be used directly after proper treatment either for artificial recharge or to create a competing source of supply.

The stochastic nature of stream flows, precipitation, natural recharge to the groundwater, and other such aspects affecting water balance in the system may play an essential role in a real system. The preliminary development here, however, is deterministic, in order to focus on the modeling procedures. A major recommendation to further improve this study's developments would be to include stochastic inputs and reduce deterministic assumptions. Actually, the modeling procedures are not restricted to deterministic systems. If the statistics of the stochastic input are known, mean value, variance, and lags should be considered inherently in the model, (Maddock, [1974]). Stream flow variations are particularly important for surface water balance and precipitation and evapo-transpiration, for groundwater balance.

We assume that for each single user, there is one aquifer cell from which he pumps his water from one or more wells. A single cell may underlie a number of stream reaches. Note that this definition of an aquifer cell is not restricted to geological or hydrological boundaries, though it may be subject to geographical, legal or political ones.

If a user operates artificial recharge facilities, these are considered aggregated at a single point inside his defined area. Water is transferred to this point from the different streams according to the recharge plan.

In the case of inelastic water demand, the economic criterion is the gross cost of water supply. Each user attempts to minimize the capital, operational, and maintenance and replacement cost of water use and artificial replenishment.

With water demand as a function of water price, the economic criterion is the net benefit obtained from water use.

The method of model superposition applied to either case may show a real advantage in the formulation process as well as in the solution strategy. The optimization problems conducted by each user are coupled to one another through the physical system. The proposed methodology enables the decoupling of these programs. A general responsive model provides each user with the following information:

1) Water levels at different operating wells during the time horizon.
2) The expected time at which drawdown at some wells will exceed casing and screening designs.
3) The quantity of water induced from the stream into an aquifer in the vicinity of the operating wells.

This information may cause the user to change his operational and design plans, in order to either reduce per unit water cost, or increase his net benefit.

These revised plans are not expected to affect total demand patterns for the inelastic case. They may, however, affect the following:

1) The operational plans of particular wells. Quantities pumped from some wells may be transferred to other wells within the aquifer cell.
2) The design plans. The user may redesign the drilling of wells and pipeline construction based on the expected water levels in the aquifer and the stream as determined by the responsive model.

If water demand is a function of water price, the total pumpage pattern and recharge plans of each user may also be subject to changes. In the following chapters, a coordination scheme is imposed on the system to provide the model with regional optimal control considerations. Each user's decisions thus become subject to input directed by the overall regional planning. It should be noted that model formulation is by no means restricted to a particular management problem. As shown later, through introducing new structural concepts in the formulation, the decomposed system functions provide an easy way for the model to successfully handle a variety of problems. Actually, in the forthcoming discussion we first analyze the proposed formulation features which may be common for different problems involving groundwater systems. Then while applying the model to two entirely different structures of case studies, the problems are still formulated and solved by the same principle, which makes use of the decomposed functions.

3.3 MODEL FORMULATION

To provide more insight into the model formulation and solution it is worthwhile to first consider the problem in the context of the optimal control of a distributed parameter system. Assume there are L users in the region. For each user there is a corresponding aquifer cell, and the ℓ^{th} user has m_ℓ wells which are located at the ℓ_{th} cell. There are U_ℓ streams traversing the ℓ^{th} cell area, from which a particular user may choose to transfer water for artificial recharge purposes to the recharge facility located in the ℓ^{th} cell area, and also to supply directly some of his water needs in that area. The ℓ^{th} user considers some or all of the following cost functions that will be discussed in detail subsequently:

1. Construction cost function:

$$Z_1^{\ell} = \int_0^T [e^{-rt} C_{\ell}(t)]dt \tag{3.1}$$

2. Pumping cost function (operation):

$$Z_2^{\ell} = \int_0^T [e^{-rt} \sum_{k_{\ell}=1}^{m_{\ell}} P_{\ell}(k_{\ell}) \cdot q_{\ell}(k_{\ell},t) \cdot h_{\ell}(k_{\ell},t)]dt \tag{3.2}$$

3. Surface water supply cost function (operation):

$$Z_3^{\ell} = \int_0^T [e^{-rt} \sum_{u=1}^{U_{\ell}} S_{\ell}(u)\ x_{\ell}(u,t)]dt \tag{3.3}$$

4. Artificial recharge cost function (operation):

$$Z_4^{\ell} = \int_0^T [e^{-rt} \sum_{u=1}^{U_{\ell}} V_{\ell}(u) \cdot v_{\ell}(u,t)]dt \tag{3.4}$$

5. Depletion of stream penalty cost function (see case study):

$$Z_5^{\ell} = \int_0^T [\ \sum_{u=1}^{U_{\ell}} Q_{\ell}(u,t) \cdot (x_{\ell}(u,t) + v_{\ell}(u,t) + f^u(\ell,t) - B_{\ell}(u,t)]dt \tag{3.5}$$

here

r	annual interest rate
$C_\ell(t)$	construction cost for water supply projects considered by user ℓ
$P_\ell(k_\ell)$	pumping cost per acre-ft/ft for the $k_\ell{}^{th}$ well
$k_\ell(k_\ell,t)$	total lift at k_ℓ time t
$S_\ell(u)$	cost per acre-ft of water supply to ℓ^{th} area from the u^{th} stream (including treatment cost)
$q_\ell(k_\ell,t)$	pumpage from the $k_\ell{}^{th}$ well
$x_\ell(u,t)$	water supply from the u^{th} stream
$v_\ell(u,t)$	recharge from the u^{th} stream
$V_\ell(u)$	recharge cost per acre-ft of water from the u^{th} stream
$Q_\ell(u,t)$	weighting function to amplify the penalty cost corresponding to the depletion of different streams traversing the ℓ^{th} area
$f^u(\ell,t)$	quantity of water induced from the u^{th} stream into the ℓ^{th} aquifer cell due to natural recharge during time period t
$B_\ell(u,t)$	upper limit for quantity of water removed from the u^{th} stream into the ℓ^{th} area by means of artificial or natural recharge and direct supply (see application to case study).

The lift $h_\ell(k_\ell,t)$ in equation (3.2) comprises the steady state lift, $H_\ell(k_\ell)$, the drawdown at k_ℓ due to pumping from wells inside ℓ, $D_\ell(k_\ell, t)$, and the drawdown at cell ℓ due to the aggregated pumping from all other cells, $\hat{D}(\ell,t)$.

Hence

$$h_\ell(k_\ell,t) = H_\ell(t) + D_\ell(k_\ell,t) + \hat{D}(\ell,t) \tag{3.6}$$

The aquifer system equations which are assumed to mathematically approximate these drawdowns are:

1. Inside the particular cell model:

$$S(\hat{x}) \frac{\partial D_\ell(\hat{x},t)}{\partial t} = \frac{\partial}{\partial \hat{x}} [T(\hat{x}) \frac{\partial}{\partial \hat{x}} D_\ell(\hat{x},t)] - \sum_{k=1}^{m_\ell} q_\ell(\hat{x}_k,t)\delta(\hat{x}-\hat{x}_k) \tag{3.7}$$

$$D_\ell(\hat{x},t) \in R \tag{3.8}$$

2. The aggregated multicell model:

$$S(\hat{x}) \frac{\partial \hat{D}(\hat{x},t)}{\partial t} = \frac{\partial}{\partial \hat{x}} [T(\hat{x}) \frac{\partial}{\partial \hat{x}} \hat{D}(\hat{x},t)] - \sum_{r=1}^{L} q_N(\hat{x}_r,t)\delta(\hat{x}-\hat{x}_r) \tag{3.9}$$

$$\hat{D}(\hat{x},t) \in \hat{R} \tag{3.10}$$

3. The steady state model:

$$\frac{\partial}{\partial x}[T(\hat{x})\frac{\partial}{\partial x}H(\hat{x})] = 0 \qquad (3.11)$$

$$H(\hat{x}) \in \bar{R} \qquad (3.12)$$

Here

$\hat{x} = (x,y)$	spatial coordinates
$S(\hat{x})$	storativity coefficient
$T(\hat{x})$	transmissivity coefficient
$\delta(\hat{x}-\hat{x}_k)$	Dirac delta function
R_ℓ	the particular ℓ^{th} cell domain, including boundary conditions.
$\bar{R}$	the particular cell domain with boundary conditions associated with steady state conditions
$\hat{R}$	the entire system (multicell) domain including boundary conditions
$q_N(\hat{x}_r,t)$	the net aggregated pumping rate from the r^{th} cell, where $q_N(\hat{x}_r,t) = q(\hat{x}_r,t) - \sum_{u=1}^{U_r} v_r(u,t)$

The flow function $f^u(\ell,t)$ in equation (3.5) comprises the stream aquifer flow function of water induced from the u^{th} stream into the ℓ^{th} aquifer cell due to pumping from inside ℓ, $\overset{*}{f}{}^u(\ell,t)$, and from the other cells, $\hat{f}^u(\ell,t)$, and the steady state flow from the u^{th} stream into the ℓ^{th} aquifer cell, I^u_ℓ. Hence

$$f^u_\ell(\ell,t) = \overset{*}{f}{}^u(\ell,t) + \hat{f}^u(\ell,t) + I^u_\ell \tag{3.13}$$

The functions in (3.13) are discussed in Phase II. They are derived respectively from the system equations (3.7) - (3.12).

At this stage we do not assume explicit solutions to the system equations (in the form of Green's functions). However in Phase II we develop the groundwork for stating the following equations:

$$\overset{*}{f}{}^u(\ell,t) = F^{*u}_\ell(\underline{q}_\ell(\hat{x}_k,t),\ D\ (x,t),\ t) \tag{3.14}$$

$$\hat{f}^u(\ell,t) = \hat{F}^u_\ell(\underline{q}(\hat{x}_r,t),\ \hat{D}(\hat{x}_r,t),\ t) \tag{3.15}$$

$$I^u_\ell = \overline{F}^u(H(\hat{x})) \tag{3.16}$$

Explicit form of the functions (3.14) - (3.16) is given in (3.43) - (3.45).

The ℓ^{th} user is evidently considering the benefits of his water use. Through the model formulation, no restriction is imposed on the particular characteristic of the water use, and benefits may be incurred by either agricultural, municipal or industrial interests.

Let $W_\ell(t)$ denote the net return per acre-ft of water supply considered by the ℓ^{th} user during time period t. Economies of scale are not considered, and the value of $W_\ell(t)$ is not affected by the quantity of supply. The benefit which the ℓ^{th} user should expect is directly related to the quantity of water he consumes:

$$W_\ell = \int_0^T [e^{-rt} W_\ell(t) \, (\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,t) + \sum_{u=1}^{U_\ell} x_\ell(u,t))]dt \qquad (3.17)$$

Actually, there are two functions which may involve economies of scale. The benefit function is practically determined by the particular user's activities, and economies of scale are introduced by construction of consuming water projects. Benefit is not an explicit function of the quantity of supply. The construction cost function (3.1), however, is eventually subject to economies of scale associated with quantity of water supply. This factor is well taken into account in recently developed capacity expansion models, Kaplan [1973]. This problem is not within the scope of this study and the construction cost function is considered only for basic analysis completeness. Notice, however, that this study is devoted to operation aspects and short-term analysis which may justify this neglect.

Under a benefit-cost analysis, Howe, [1971], the ℓ^{th} user is interested in maximizing the criterion functional $\hat{Z}_\ell$:

$$\max_{(\underline{q},\underline{x},\underline{v})_\ell} \quad (\hat{Z}_\ell = W_\ell - \sum_{p=1}^{5} Z_p^\ell) \tag{3.18}$$

where W_ℓ is given by (3.17) and Z_p^ℓ, p=1,...,5, are given by (3.1) - (3.5).

In addition to the system equations (3.6) - (3.16), which must be satisfied by the optimal solution to (3.18) there are restrictions (physical, economic or others) to account for:

1. Minimum water requirements must be met:

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,t) + \sum_{u=1}^{U_\ell} x_\ell(u,t) \geq R_\ell(t) \qquad t \in [0,T] \tag{3.19}$$

2. Drawdown must not exceed designs:

$$h_\ell(k_\ell,t) \leq h_{\ell max}(k_\ell) \qquad t \in [0,T] \quad k_\ell=1,\ldots,m_\ell \tag{3.20}$$

3. Pumping capacity must be restricted:

$$q_\ell(k_\ell,t) \leq Q_{\ell max}(k_\ell) \qquad t \in [0,T] \quad k_\ell=1,\ldots,m_\ell \tag{3.21}$$

4. Recharge facility capacity must be constrained:

$$\sum_{u=1}^{U_\ell} v_\ell(u,t) \leq v_{\ell max} \qquad t \in [0,T] \tag{3.22}$$

5. Surface water supply must have an upper limit:

$$x_\ell(u,t) \leq x_{\ell max}(u) \qquad t \in [0,T] \quad u=1,\ldots,U_\ell \tag{3.23}$$

6. Infiltrating rate limit must be constrained:

$$f^u(\ell,t) \leq Q^u_{INF,\ell} \qquad t \in [0,T] \quad u=1,\ldots,U_\ell \tag{3.24}$$

here

$R_\ell(t)$	minimal water requirements function
$h_{\ell max}(k_\ell)$	maximum lift allowed at the k_ℓ^{th} well
$Q_{\ell max}(k_\ell)$	upper limit for pumping from k_ℓ
$v_{\ell max}$	recharge facility capacity limit
$x_{\ell max}(u)$	surface water supply system from the u^{th} stream capacity limit
$Q^u_{INF,\ell}$	maximum infiltrating rate from the u^{th} stream into the ℓ^{th} cell

The mathematical model defined by (3.1) - (3.24) constitutes an optimal control problem in a distributed parameter system. Evidently in its present form the classical control is inadequate for solving the optimal control policy. Fortunately, the application of numerical techniques based on certain assumptions reduces the model to a form where well-known techniques from systems engineering are applicable for optimally solving the system.

In accordance with what we stated in Section 3.1, a better insight into the control problem is achieved by analyzing the system using methodologies from the field of optimal control. A main source of complication which is introduced to the original problem is caused by the distributed parameter system equations and the fact that the waterhead distribution must be coupled with the control variables. Therefore, prior to solving the original problem, a simplified case is considered. Conserving the main features of the original problem, it should provide the analytical tool for studying the nature of the problem and its solution.

3.4 A SIMPLIFIED CASE FOR MANAGEMENT CONTROL STUDY

In the following we develop the ground for stating a necessary condition for optimal solution to the problem formulated in (3.1) to (3.24).

Theorem: A necessary condition for the control problem of a distributed parameter groundwater system, as formulated in (3.1) through (3.24) so as to constitute an optimal control solution, is that the Green's functions of the systems in (3.7) through (3.12) should be in positive times and the constraints in (3.19) through (3.24) should be a convex set.

Proof:

Consider a single aquifer cell which is described by the following system equations:

$$S \frac{\partial D(x,t)}{\partial t} = T \frac{\partial^2 D(x,t)}{\partial x^2} - \sum_{k=1}^{M} q(x_k,t)\delta(x-x_k) \qquad \begin{array}{l} x \in [0,L) \\ t \in [0,1] \end{array} \tag{3.25}$$

and boundary conditions:

$$D(x,0) = g(x) \tag{3.26}$$

$$D(0,t) = D(L,t) = 0 \tag{3.27}$$

here S and T are storage and transmissivity coefficients, respectively, in the homogeneous one-dimensional space. $D(x,t)$ is the drawdown function, $q(x_k,t)$ is the pumpage from a well located at x_k and there are M wells in the field. δ is the Dirac delta function. $g(x)$ is a known function of initial head distribution. The mathematical model defined by (3.25) - (3.27) has the solution, [Roach, 1970]:

$$D(x_k,t) = \sum_{j=1}^{M} \int_{0}^{t} G(x_k,x_j,t-\tau)q(x_j,\tau)d\tau \qquad t \in [0,1] \tag{3.28}$$

where G is the Green's Function which is explicitly derived for a given $g(x)$ in terms of the system's eigen-values and eigen-functions, (see Appendix A, Phase II).

Define the planning time horizon T and let [0,1] in (3.25) comprise a unit time step, so that there are exactly N such time steps in the horizon, n=1,...,N. The pumping from a well at x_k, $q(x_k,t)$ is assumed to comprise a series over time of discharge rates, where the rate is constant during each single time step, but may vary from time step to time step. Hence

$$q(x_k,t) = q(k,n), \quad n=1,\ldots,N$$

Considering only pumping from wells, and no recharge or surface water supply options, the performance criterion function is:

$$Z = \int_0^T [\hat{P}(t)\underline{q}(t)\underline{D}^T(t)]dt = \sum_{n=1}^{N} \int_{t=n-1}^{n} [\hat{P}(t)\underline{q}(n)\underline{D}^T(t)]dt \quad (3.29)$$

where $\hat{P}(t) = e^{-rt}P(t)$ and r is the discount rate. Substitute ((3.28) into (3.29) to obtain

$$Z = \sum_{n=1}^{N}\sum_{k=1}^{M} \int_{n-1}^{n} [\hat{P}(t)q(k,n) \sum_{j=1}^{M} \int_{n-1}^{t} G_n(k,j,t-\tau)q(j,n)d\tau]dt \quad (3.30)$$

$G_n(k,j,t-\tau)$ is the Green's function for the system equations (3.25) - (3.27) where $t \in [n-1,n]$ and $g(x) = D(x,n-1)$ (3.31) is the initial condition.

In a compact form, (3.30) becomes:

$$Z = \sum_{n=1}^{N} \int_{n-1}^{n} [\hat{P}(t)\underline{q}(n)\underline{B}_n(t)\underline{q}^T(n)]dt$$

where $$\underline{B}_n(t) = \int_{n-1}^{t} \underline{G}_n(\tau)d\tau \qquad t \in [n-1,n] \tag{3.32}$$

$\underline{G}_n(t)$ is a matrix of the Green's function whose elements $G_n(k,j,t-\tau)$ state the response at k due to unit pumping at j for the n^{th} time period. $\underline{B}_n(t)$ is a matrix whose elements are

$$B_n(k,j,t) = \int_{n-1}^{t} G_n(k,j,t-\tau)d\tau$$

Finally, as $\underline{q}(n)$ is a time invariant function for each $n, n=1,\ldots,N$:

$$Z = \sum_{n=1}^{N} \underline{q}(n) \cdot \int_{n-1}^{n} \hat{P}(t)\, \underline{B}_n(t)dt \cdot \underline{q}^T(n)$$

$$= \sum_{n=1}^{N} \underline{q}(n) \cdot \underline{B}(n) \cdot \underline{q}^T(n) \tag{3.33}$$

where

$$\underline{B}(n) = \int_{n-1}^{n} \hat{P}(t)\, \underline{B}_n(t)\, dt$$

Equation (3.33) states that the criterion function (3.29) comprises the summation of n decoupled quadratic terms, each depending on the system solution at a particular time period n, $n=1,\dots,N$. Necessary and sufficient conditions for the criterion function (3.29) to constitute a unique optimal control solution for a convex constraints set is that $\underline{B}(n), n=1,\dots,N$ should be positive definite matrices, (Hadley, [1964], Bryson and Ho, [1969]).

To understand the immediate application of this result to the management control problem, we now investigate the physical meaning of the $\underline{B}(n)$ matrices. The criterion function essentially consists of a discounted multiplication of flows and the associated lifts. Equating equations (3.29) and (3.33) yields the following:

$$Z = \sum_{n=1}^{N} \underline{q}(n) \cdot \underline{B}(n) \cdot \underline{q}^T(n) = \sum_{n=1}^{N} \hat{P}(n) \cdot \underline{q}(n) \cdot \underline{D}(n) \quad (3.34)$$

Here $\hat{P}(n)$ is the discount factor for the n^{th} time step, and $\underline{D}(n)$ is the vector of water head drawdown in the pumping wells at the end of the n^{th} period. But $\underline{D}(n)$ is also the solution to the system equation (3.25) for $t \in [0,n]$ and the initial condition $\underline{D}(x,0) = g(x)$, and is given by:

$$\underline{D}(n) = \int_0^n \underline{G}(\tau)\underline{q}(\tau)d\tau \quad (3.35)$$

where $\underline{G}$ is the Green's function defined for $t \in [0,n)$ and there are n time steps in t. Substitute (3.35) into (3.34) to obtain:

$$Z = \sum_{n=1}^{N} \underline{q}(n)\ \underline{B}(n)\ \underline{q}^{T}(n) = \sum_{n=1}^{N} \hat{P}(n)\ \underline{q}(n) \int_{0}^{n} \underline{G}(\tau)\ \underline{q}(\tau) d\tau \quad (3.36)$$

Equation (3.36) implies, that for $\underline{B}(n)$ to be a positive definite matrix, the integral on the right-hand side of (3.36) should be positive for all n, given $\underline{q}(t)$ positive function. This is true provided $\underline{G}(t)$, the Green's matrix for the system's mathematical model, is positive for all t. This last conclusion is applicable for stating the necessary and sufficient conditions for optimality in the simplified optimal control case. However for the original problem these conditions may not be sufficient but necessary, as more elements other than pumping wells are considered. By this we conclude the proof. The theorem is simply stating that the management control model can be applied to systems which do not contain certain irregularities. In this sense an irregularity means that it is possible under a certain circumstance that imposed pumpage will induce a negative drawdown at some point in the aquifer. Such a situation would be very rare.

Our next step is to solve the original distributed parameter system optimal control by doing a numerical analysis.

3.5 A NUMERICAL SOLUTION PROCEDURE

3.5.1 Model formulation

There are two basic concepts which we use for properly reformulating the management control as was discussed in Section 3.3. First, discretizing the time dimension allows for converting the time integrals into summations over a series of time steps. The second concept used is the one developed in Phase II of this study. It assumes the existence of Green's functions for the systems which are modeled by equations (3.7) through (3.12). An aquifer simulation model is used for determining these Green's functions for certain points in time and space. Consequently, fraction algebraic functions are derived to approximate infiltration rates through stream beds. The superposition of both the Green's functions (β's, γ's) and the fraction functions (ϕ's, ψ's) is applied. A detailed discussion of these functions is in Part II of this report. Resulting from these two concepts is the following quadratic program:

$$\max_{(\underline{q},\underline{x},\underline{v})_\ell} \left\{ \hat{Z}_\ell = \sum_{n=1}^{T} (1+r)^{-n} \cdot W_\ell(n)\left[\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + \sum_{u=1}^{U_\ell} x_\ell(u,n)\right] \right.$$

$$- \sum_{n=1}^{T} (1+r)^{-n} C_\ell(n)$$

$$- \sum_{n=1}^{T} (1+r)^{-n}\left[\sum_{k_\ell=1}^{m_\ell} P_\ell(k_\ell) \cdot q_\ell(k_\ell,n)\cdot h_\ell(k_\ell,n)\right]$$

$$- \sum_{n=1}^{T} (1+r)^{-n} \left[\sum_{u=1}^{U_\ell} S_\ell(u) \cdot x_\ell(u,n)\right]$$

$$- \sum_{n=1}^{T} (1+r)^{-n} \left[\sum_{u=1}^{U_\ell} V_\ell(u) \cdot v_\ell(u,n)\right]$$

$$- \sum_{n=1}^{T} \sum_{u=1}^{U_\ell} Q_\ell(u,n) \cdot [x_\ell(u,n) + v_\ell(u,n)$$

$$\left. + f^u(\ell,n) - B_\ell(u,n)] \right\} \tag{3.37}$$

With the system's equations in the form of algebraic technological functions (A.T.F.):

$$h_\ell(k_\ell,n) = H_\ell(k_\ell) + D_\ell(k_\ell,n) + \hat{D}(\ell,n) \tag{3.38}$$

where $D_\ell(K_\ell,n) = \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} [\beta_\ell(k_\ell,j,n-i+1) q_\ell(j,i)]$

$$-\sum_{i=1}^{n} [\beta_\ell(k_\ell,v_\ell,n-i+1).v_\ell(u,i)] \tag{3.39}$$

$$\hat{D}(\ell,n) = \sum_{\substack{r=1\\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1)\,[q(r,i)-v(r,i)] \tag{3.40}$$

$$q(r,i) = \sum_{k_r=1}^{m_r} q_r(k_r,i) \tag{3.41}$$

$$v(r,i) = \sum_{u=1}^{u_r} v_r(u,i) \tag{3.42}$$

and the stream-aquifer flow functions:

$$\overset{u}{f}(\ell,n) = \overset{*}{f}{}^{u}(\ell,n) + \hat{f}^{u}(\ell,n) + I_\ell^u \tag{3.43}$$

where

$$\overset{*}{f}{}^{u}(\ell,n) = \sum_{k_\ell=1}^{m_\ell} \sum_{i=1}^{n} \overset{u}{\phi}_\ell(k_\ell,n-i+1).q_\ell(k_\ell,i) \tag{3.44}$$

$$\hat{f}^{u}(\ell,n) = \sum_{\substack{r=1\\ r\neq\ell}}^{L} \sum_{i=1}^{n} \overset{u}{\psi}_\ell(r,n-i+1)\,[q(r,i)-v(r,i)] \tag{3.45}$$

The notations used in (3.37)-(3.45) are essentially the same as those used for the original distributed parameter control problem formulated in Section 3.3. The discretization of time t into n time periods provided the above objective function formulation. For the system equations, the following terms were used based on the existence of the Green's functions:

$\beta_\ell(k_\ell,j,n-i-1)$ is the algebraic technological term relating the drawdown at the k_ℓ-th well to the pumping of one unit of water from the j-th well during the i-th period. Both k_ℓ and j are located at the ℓ-th cell.

$\gamma(\ell,r,n-i+1)$ is the algebraic technological term relating the average drawdown at the ℓ-th cell to aggregated pumping of one unit of water at the r-th cell, during the i-th period.

$\phi_\ell^u(k_\ell,n-i+1)$ is the quantity of water induced from the u-th stream into the ℓ-th cell during the n-th period due to one unit of pumping at the k_ℓ-th well during the i-th period.

$\psi_\ell^u(r,n-i+1)$ is the quantity of water induced from the u-th stream into the ℓ-th cell during the n-th period due to one unit of pumping at the r-th cell during the i-th period.

I_ℓ^u is the quantity of water induced from the u-th stream into the ℓ-th cell during one time period with no imposed pumpage and the system in steady state.

The system's constraints follow the same order as the constraints set in the original model:

$$\sum_{k_\ell=1}^{m_\ell} q_\ell(k_\ell,n) + \sum_{u=1}^{U_\ell} x_\ell(u,n) \geq R_\ell(n) \qquad n=1,\ldots,T \qquad (3.46)$$

$$h_\ell(k_\ell,n) \leq h_{\ell max}(k_\ell) \qquad n=1,\ldots,T \quad k_\ell=1,\ldots m_\ell \qquad (3.47)$$

$$q_\ell(k_\ell,n) \leq Q_{\ell max}(k_\ell) \qquad n=1,\ldots,T \quad k_\ell=1,\ldots,m_\ell \qquad (3.48)$$

$$\sum_{u=1}^{U_\ell} v_\ell(u,n) \leq v_{\ell max} \qquad n=1,\ldots,T \qquad (3.49)$$

$$x_\ell(u,n) \leq x_{\ell max}(u) \qquad n=1,\ldots,T \quad u=1,\ldots,U_\ell \qquad (3.50)$$

$$f^u(\ell,n) \leq Q^u_{INF,\ell} \qquad n=1,\ldots,T \quad u=1,\ldots,U_\ell \qquad (3.51)$$

3.5.2 Solution Strategies

The quadratic optimization program stated in (3.37) through (3.51) is considered solely by the ℓ-th user in the basin. However, there are other water users in the area, and up to L such distinct optimization programs may be respectively performed and each would correspond to a single user. Each individual program can be solved provided it is decoupled from other activities which are not under the ℓ-th user direct control. The L programs are coupled through the physical system responses, including the $\hat{D}(\ell,n)$ and $f^u(\ell,n)$ functions relating the system effect on the ℓ-th user from pumpage imposed in other parts of the hydrologic system by other users. In addition, stream balance considerations, such as the term $B_\ell(u,n)$, couple the systems' operations which are performed by all users.

1. The coupling through the term $\hat{D}(\ell,n)$.

In equation (3.40) we represented $\hat{D}(\ell,n)$ explicitly:

$$\hat{D}(\ell,n) = \sum_{\substack{r=1 \\ r\neq\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1)\ [q(r,i) - v(r,i)] \tag{3.52}$$

$q(r,i)$, $v(r,i)$, are the aggregated pumpage and artificial recharge, respectively, which are considered by users for different cells. Once these values are specified, the solution for $\hat{D}(\ell,n)$ is explicitly given in (3.52).

2. The coupling through the term $f^u(\ell,n)$

In equation (7.43) $f^u(\ell,n)$ was defined:

$$f^u(\ell,n)=\overset{*}{f}{}^u(\ell,n) + \hat{f}^u(\ell,n) + I_\ell^u \quad (3.53)$$

and $$\hat{f}^u(\ell,n) = \sum_{\substack{r=1\\ r\neq\ell}}^{L} \sum_{i=1}^{n} \psi_\ell^u(r,n-i+1)\,[q(r,i)-v(r,i)] \quad (3.54)$$

The same arguments are used here for the coupling term $\hat{f}^u(\ell,n)$ where specification of $q(r,i)$ and $v(r,i)$ provides explicitly the value of $f^u(\ell,n)$.

3. The coupling through the term $B_\ell(u,n)$.

The value of the term $B_\ell(u,n)$ should be assigned externally to the optimum control problem being considered by a particular user. The stream balance evidently concerns each user but is affected by all users' operations and by other things not controlled by any of them such as upstream inflow. It is therefore assumed that stream balance terms like $B_\ell(u,n)$ are specified for each user for each problem setting. In the following chapters at least one possible approach is presented to assign stream balance terms to each user according to an external consideration set.

Some of the conceptual solution strategies are illustrated by analyzing two case studies.

3.6 A QUADRATIC PROGRAM MODEL

This section is concerned with using a standard quadratic programming solution for this study's model. A modification of the procedure developed by Wolfe, [1959], and programmed by Bates, [Bates H.], is presented. A listing of the source program is in Kuster and Mize, [1973]. Originally, the procedure suggested by Wolfe, [1959] is for the following problem:

PROBLEM A:

$$\min_{\underline{x}} \quad Z = \underline{P}\,\underline{x} + 1/2\,\underline{x}^T\,\underline{C}\,\underline{x}$$

$$\underline{A}\,\underline{x} \le \underline{b} \tag{3.55}$$

$$\underline{x} \ge \underline{0}$$

where

$$\underline{x} = (x_1, x_2, \ldots, x_n)^T$$

$$\underline{P} = (P_1, P_2, \ldots, P_n)$$

$$\underline{b} = (b_1, b_2, \ldots, b_m)^T$$

$$\underline{A} = \begin{bmatrix} a_{11} & a_{1n} \\ a_{m1} & a_{mm} \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} c_{11} & c_{1n} \\ c_{n1} & c_{nn} \end{bmatrix}$$

The requirements for problem 1 to obtain a solution via the proposed procedure are:

a) The matrix $\underline{C}$ is assumed positive definite and symmetric.

b) The constraints are assumed to be of the form:

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j \qquad j=1,\ldots,m$$

and all b_j are non-negative.

If these requirements are fulfilled, a solution is warranted, Hadley, [1964]. The problem formulation as in section 3.5.1 reduces to the compact vector form of problem B:

PROBLEM B:

$$\begin{aligned}
&\underset{\underline{x}}{\text{Minimize}} \quad (Z = \underline{P}\,\underline{x} + 1/2\,\underline{x}^T \underline{C}\,\underline{x}) \\
&\underline{A}^1 \underline{x} \leq \underline{b}^1 \qquad \underline{b}^1 \geq 0 \\
&\underline{A}^2 \underline{x} \geq \underline{b}^2 \qquad \underline{b}^2 \geq 0 \\
&\underline{x} \geq 0
\end{aligned} \tag{3.56}$$

where

$$\underline{A}^1 = \begin{bmatrix} a^1_{11} & a^1_{1n} \\ a^1_{P1} & a^1_{Pn} \end{bmatrix} \qquad \underline{A}^2 = \begin{bmatrix} a^2_{11} & a^2_{1n} \\ a^2_{q1} & a^2_{qn} \end{bmatrix}$$

$$\underline{b}^1 = (b_1^1,\ldots,b_p^1)^T \qquad b_i^1 \geq 0 \qquad i=1,\ldots,P$$

$$\underline{b}^2 = (b_1^2,\ldots,b_q^2)^T \qquad b_i^2 > 0 \qquad i=1,\ldots,q$$

Unfortunately, the constraints $\underline{A}^2\,\underline{x} \geq \underline{b}^2$ contradict requirements (b) for the application of the Wolfe algorithm. The following technique is suggested to overcome this problem:

$$\text{Define vectors} \quad \underline{Y} = (y_1,y_2,\ldots,y_q)^T$$

$$\underline{x}^1 = (x_1^1,x_2^1,\ldots,x_n^1)$$

$$\underline{m} = (m_1,\ldots,m_q)$$

x_i^1, y_i are decision variables, m_i is a non-negative and yet unspecified number. A new quadratic programming model is defined.

PROBLEM C:

$$\underset{\underline{x}',\underline{Y}}{\text{Minimize}} \; (Z_m = \underline{P}\,\underline{x}^1 + 1/2\,\underline{x}^{1T}\,\underline{C}\,\underline{x}^1 - \underline{m}\,\underline{Y})$$

$$\underline{A}^1\,\underline{x}^1 \leq \underline{b}^1$$

$$\underline{Y} - \underline{A}^2\,\underline{x}^1 \leq \underline{0} \qquad \underline{x}^1 \geq 0 \qquad \underline{Y} \geq 0$$

$$\underline{Y} \leq \underline{b}^2 \tag{3.57}$$

Theorem: If the problem B poses an optimal solution Z^* for $\underline{x}^* = (x_1^*,\ldots,x_n^*)^T$, then Z_m^* is the optimal solution for Problem C with $\underline{x}^1 \equiv \underline{x}^{1*}$ and $\underline{Y}^* \equiv \underline{b}^2$ where $Z_m^* = Z^* - \underline{m}\,\underline{b}^2$.

Proof: One should observe, that if in Problem C the vector of variables $\underline{Y}$ is set to $\underline{Y} = \underline{b}^2$ so that $y_i = b_i^2$, $i=1,\ldots,q,$ then Problem C is reduced to the original Problem B with the objective function value differing in a constant scalar $\underline{m}\,\underline{b}^2$. Hence, if we prove that $\underline{Y} = \underline{b}^2 = \underline{Y}^*$ for Z_m^*, then the optimal solution of C for $\underline{x}^1 = \underline{x}^{1*}$ coincides with the optimal solution of B for $\underline{x} = \underline{x}^*$ and $x^{1*} = \underline{x}^*$.

To prove $\underline{Y}^* = \underline{b}^2$ we apply the Kuhn-Tucker (Kuhn and Tucker, [1961]) necessary conditions for optimality to both problems B and C.

$$\text{Let}\quad \underline{\lambda}^1 = (\lambda_1^1, \lambda_2^1, \ldots, \lambda_p^1)^T$$

$$\underline{\lambda}^2 = (\lambda_1^2, \lambda_2^2, \ldots, \lambda_q^2)^T$$

be the Lagrange multipliers corresponding to the two sets of constraints $\underline{A}\,\underline{x} - \underline{b}^1 \leq \underline{0}$, $\underline{b}^2 - \underline{A}^2\,\underline{x} \leq \underline{0}$, respectively in Problem B and $\underline{A}^1\,\underline{x}^1 - \underline{b}^1 \leq \underline{0}$, $\underline{Y} - \underline{A}^2\,\underline{x}^1 \leq \underline{0}$ in Problem C. Let $\underline{\lambda}^3 = (\lambda_1^3, \ldots, \lambda_q^3)^T$ be the Lagrange multipliers corresponding to the sets of constraints $\underline{Y} - \underline{b}^2 \leq \underline{0}$ in Problem C, thus the application of the Kuhn-Tucker conditions to Problem B yields:

$$1)\quad x_i\Big(P_i + \sum_{j=1}^{n} C_{ij}\, x_j + \sum_{j=1}^{p} a_{ij}^1\, \lambda_j^1 - \sum_{j=1}^{q} a_{ji}^2\, \lambda_j^2\Big) = 0 \qquad i = 1,\ldots,n$$

$$2)\quad x_i \geq 0 \qquad i = 1,\ldots,n$$

3) $\underline{P} + \underline{C}\ \underline{x} + \underline{A}^{1T}\ \underline{\lambda}^{1} - \underline{A}^{2T}\ \underline{\lambda}^{2} \geq \underline{0}$

4) $\lambda_i^1\left(\sum_{j=1}^{n} a_{ij}^1\ x_j - b_j^1\right) = 0 \qquad i=1,\ldots,P$

5) $\lambda_i^1 \geq 0 \qquad i=1,\ldots,P$

6) $\underline{A}^1\ \underline{x} - \underline{b}^1 \leq \underline{0}$

7) $\lambda_i^2\left(b_i^2 - \sum_{j=1}^{n} a_{ij}^2\ x_j\right) = 0 \qquad i=1,\ldots,q$

8) $\lambda_i^2 \geq 0 \qquad i=1,\ldots,q$

9) $\underline{b}^2 - \underline{A}^2\ \underline{x} \leq \underline{0}$ (3.58)

Applying the same conditions to problem C yields:

1) $x_i^1\left(P_i + \sum_{j=1}^{n} C_{ij}\ x_j^1 + \sum_{j=1}^{P} a_{ji}^1\ \lambda_j^1 - \sum_{j=1}^{q} a_{ji}^2\ \lambda_j^2\right) = 0$

$i = 1,\ldots,n$

2) $x_i^1 \geq 0 \qquad i = 1,\ldots,n$

3) $\underline{P} + \underline{C}\ \underline{x}^1 + \underline{A}^{1T}\ \underline{\lambda}^{1} - \underline{A}^{2T}\ \underline{\lambda}^{2} \geq \underline{0}$

4) $\lambda_i^1\left(\sum_{j=1}^{n} a_{ij}^1\ x_j^1 - b_j^1\right) = 0 \qquad i=1,\ldots,P$

5) $\lambda_i^1 \geq 0 \qquad i = 1,\ldots,P$

6) $\underline{A}^1\,\underline{x}^1 - \underline{b}^1 \leq \underline{0}$

7) $\lambda_i^2(y_i - \sum_{j=1}^{n} a_{ij}^2\, x_j^1) = 0 \qquad i=1,\ldots,q$

8) $\lambda_i^2 \geq 0 \qquad i = 1,\ldots,q$

9) $\underline{Y} - \underline{A}^2\,\underline{x}^1 \leq \underline{0}$

10) $\lambda_i^3(y_i - b_i^2) = 0$

11) $\lambda_i^3 \geq 0 \qquad i = 1,\ldots,q$

12) $\underline{Y} - \underline{b}^2 \leq \underline{0}$

13) $y_i(-m_i + \lambda_i^2 + \lambda_i^3) = 0 \qquad i=1,\ldots,q$

14) $y_i \geq 0 \qquad i = 1,\ldots,q$

15) $-\underline{m}^T + \underline{\lambda}^2 + \underline{\lambda}^3 \geq \underline{0} \qquad (3.59)$

Condition (10) in Problem C states that either $y_i = b_i^2$ or $\lambda_i^3 = 0$, $i = 1,\ldots,q$. Let $y_i = b_i^2$, $i = 1,\ldots,q$ and substitute into equations 7 and 9 in Problem C. The set of equations 1-9 in Problem C is identical to the equations which result from applying the Kuhn-Tucker conditions to Problem B. Assuming that Problem B constitutes a solution then this same solution must hold for the subset of equations 1-9 in Problem C. In order that such a solution holds for the entire set of equations in Problem C, equations 10-15 should also be satisfied. The condition $y_i = b_i^2$ satisfies both equations 10 and 12. Given b_i^2 positive, then equation 13 states $y_i > 0 \rightarrow -m_i + \lambda_i^2 + \lambda_i^3 = 0$. Condition 11 states that $\lambda_i^3 \geq 0$ and hence $m_i - \lambda_i^2 \geq 0$, or $m_i \geq \lambda_i^2$. This should also satisfy condition 15. We may now conclude, that if $\underline{m}$ is set to $m_i \geq \lambda_i^2$, $i = 1,\ldots,q$, the entire set of conditions is satisfied provided Problem B has a solution. This implies that $\underline{Y}^* = \underline{b}^2$ is the optimal value of $\underline{Y}$, and the proof is concluded.

In the following chapters this proposed modification is actually used and provides the utilization of the standard quadratic program of Problem A.

CHAPTER 4

A CASE STUDY
APPLICATION OF THE MANAGEMENT CONTROL MODEL TO THE FAIRFIELD-NEW BALTIMORE AREA

4.1 INTRODUCTION

A detailed description of the Fairfield-New Baltimore area in southwestern Ohio is given in Phases I and II, Haimes [1973,1974], and in Chapter 2 of this report. A simulation procedure which is developed in Chapter 1 was applied to the aquifer underlying the area. Consequently, Algebraic Technological Functions (A.T.F.) which are developed in Phase II to relate drawdown to pumping from wells was constructed for wells located at the studied area. Flow fraction functions between streams and aquifer relating to well pumpage were also determined for application to the particular area. The management control model introduced in Chapter 3 comprises in its structure and its formulation most of what was discussed in Phase II for coupling the physical system with the desired control scheme. Thus, the functions determined throughout this study are now available for coupling the Fairfield-New Baltimore system with any imposed control scheme. The water resources in the Fairfield-New Baltimore area are under the supervision of the Miami Conservancy District (M.C.D.) and the U.S. Geological Survey, (U.S.G.S.). However, neither the M.C.D. nor any other authority has the jurisdiction to impose a regional policy for water resources development, (Speiker, [1968], Plummer, [1974]). As a result, water users are free to choose their own policies for developing their water supply systems, and only a few restrictions are imposed with respect to water quality, the Clean Water Act, [1972], and Water Rights (Speiker, [1968], Cincinnati Well Field Case). The management model in Chapter 3 may be reduced to handle the Fairfield-New Baltimore case study. Actually, the model does not assume any administrative coordination between activities of individual water users in the area. The only connection between these activities is essentially their common need to take into account the physical system's response. Each user can do this provided his own optimal performance is subject to feed-in of information of others' activities. Such an information flow is actually available from the proposed management formulation using the response functions hierarchy.

We have identified five major users in the Fairfield-New Baltimore area (Plummer, [1974]), Figure 4.1:

1) American Cyanamid + Fisher Body (Cell 2)
2) Hamilton South Field + Fairfield (Cell 4)
3) Southwestern Ohio (Cell 5a)
4) Cincinnati (Cell 5b)
5) U.S. Atomic Energy Commission (Cell 6)

Others use relatively small amounts of water and can be ignored for our purposes here.

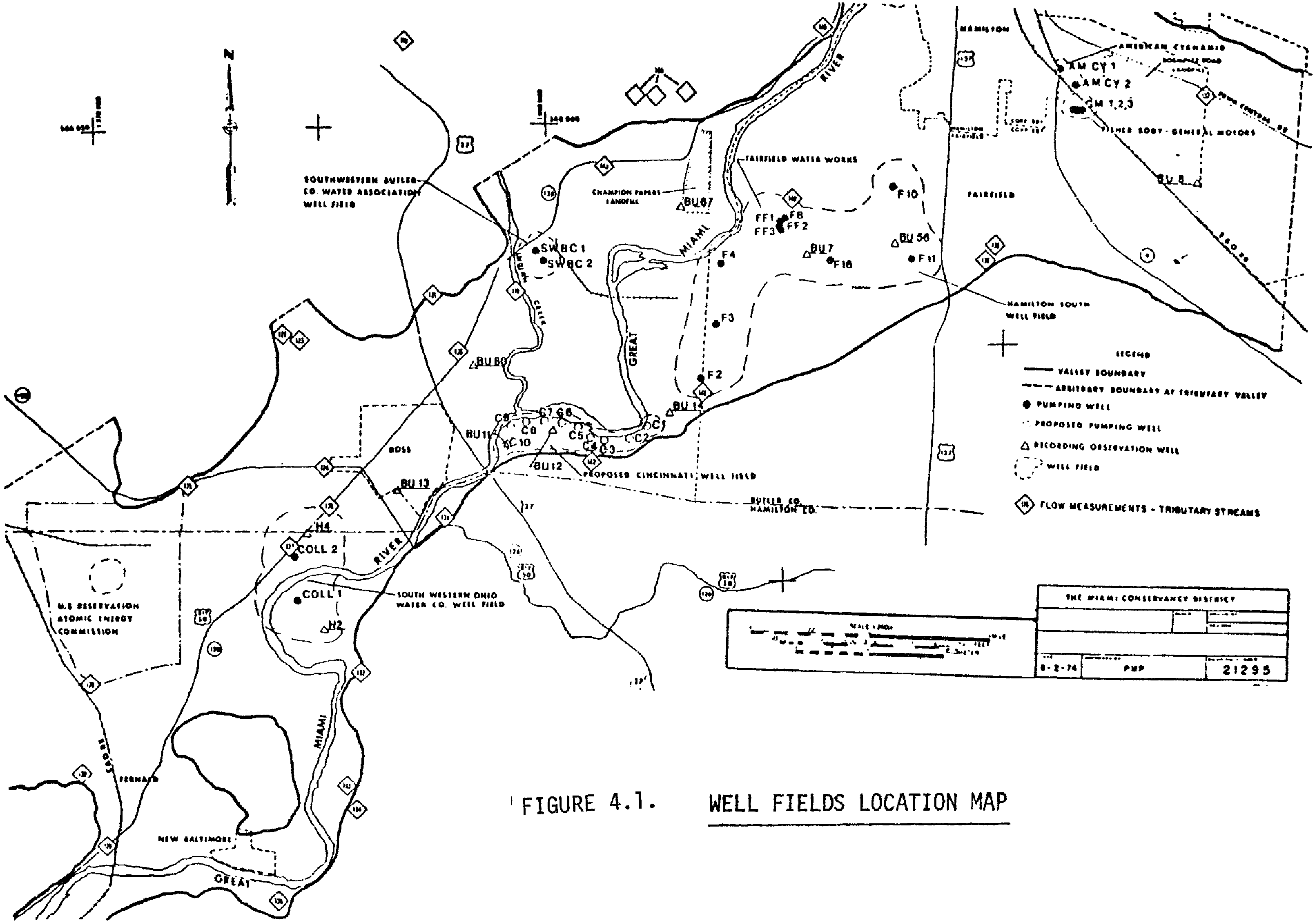


FIGURE 4.1. WELL FIELDS LOCATION MAP

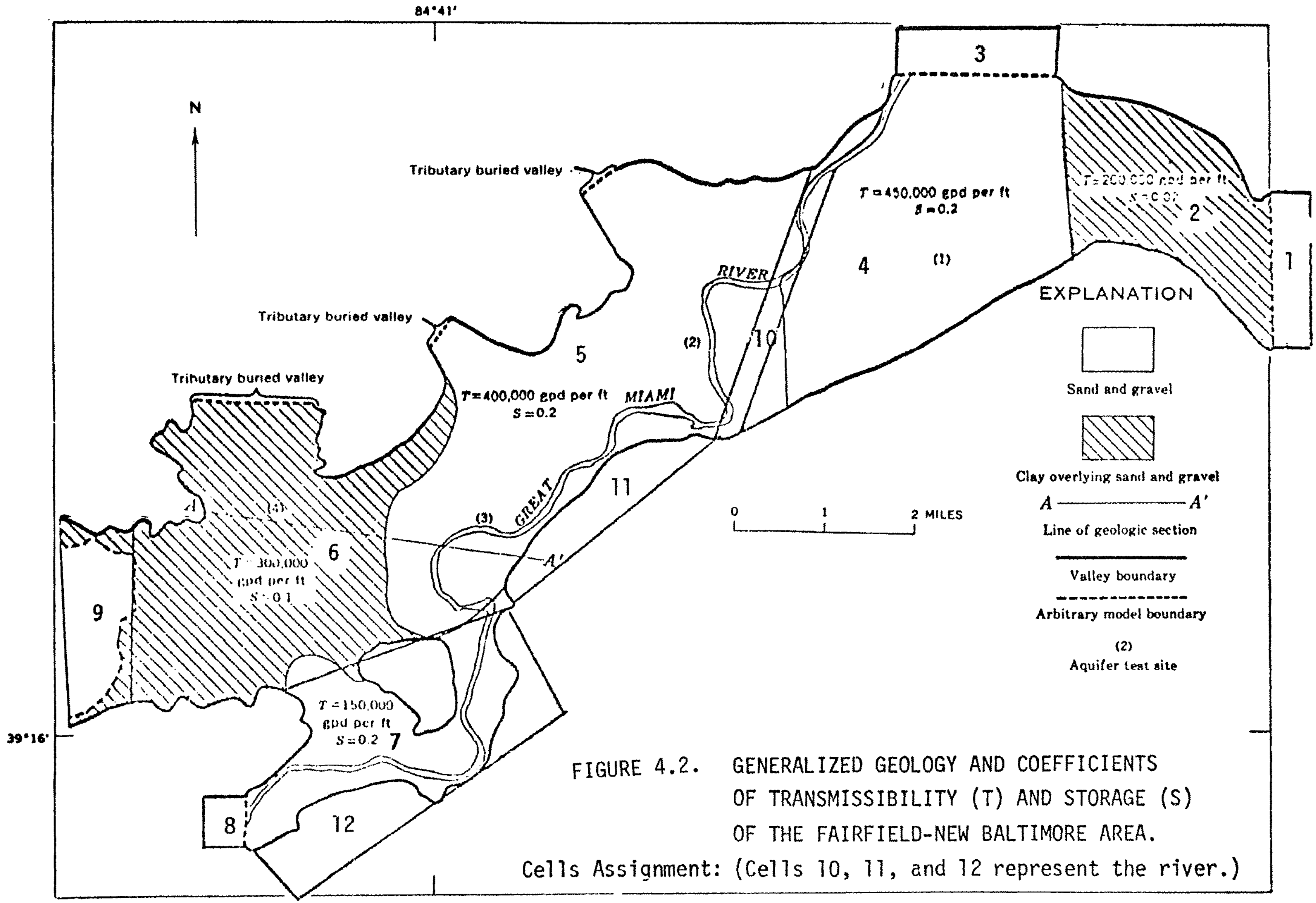


FIGURE 4.2. GENERALIZED GEOLOGY AND COEFFICIENTS OF TRANSMISSIBILITY (T) AND STORAGE (S) OF THE FAIRFIELD-NEW BALTIMORE AREA.

Cells Assignment: (Cells 10, 11, and 12 represent the river.)

Water needs in the Fairfield-New Baltimore area are classified for municipal and industrial use. At present all water requirements are met by groundwater from operating wells. No direct supply from streams is yet considered, due to water availability from the aquifer and quality restrictions on surface water. Also there is no need for artificial recharge, therefore, it has not been introduced. Information is available for identifying the physical system. Also available are some projections of future water needs. It is assumed that these needs will be inelastic and that users will not be concerned about cost of water, only its availability.

The main goal of applying the management model to this case is to come up with a prediction tool to evaluate water use activities and the system's response to them. The resulting policy may be acceptable to the water users because it assumes that they all will seek an optimal operation policy. It should point out some of the most critical developments in the system while supply is increasing, and may probably initiate the desire for a coordinated system providing improved exploitation of the water resources.

4.2 APPLICATION TO THE FAIRFIELD-NEW BALTIMORE CURRENT ADMINISTRATIVE STRUCTURE

Unfortunately, the current situation in the Fairfield-New Baltimore area includes only part of the options accounted for in the management model in Chapter 3. Actually, we do not propose that the general model be applied only to cases where all the options encompassed by the model pertain. In the following, only a certain part of the general model formulation is applied to the actual case as defined by the Fairfield-New Baltimore area. We utilize the following information:

Table 4.1 summarizes the projections of water requirements for 1974-1983, (Spieker, [1968], Plummer, [1974]). Table 4.2 tabulates the algebraic technological functions (A.T.F.) relating drawdowns in wells to aggregated pumpage, under various boundary conditions along the stream reaches. More detailed data are available for Hamilton South Field (Cell 4). Table 4.3 tabulates the A.T.F. functions corresponding to three wells in that field. Functions of flows between stream and aquifer related to pumping from cells are tabulated in Table 4.4. In Table 4.5 maximum infiltration rates from stream reaches into the aquifer are listed, (based on Spieker, [1968]).

TABLE 4.1

WATER REQUIREMENTS PROJECTIONS IN THE FAIRFIELD-NEW BALTIMORE AREA

(Figures are given in acre-ft/day)

Year	Cell					
	2	4	5a	5b	5a & 5b	6
1974	1.5	30.6	55.7		55.7	3.
1975	1.6	31.2	57.2		57.2	3.
1976	1.7	31.8	58.7	122.	180.7	3.
1977	1.8	32.4	60.2	122.	182.2	3.
1978	1.9	33.0	61.7	122.	183.7	3.
1979	2.0	33.6	63.2	122.	185.2	3.
1980	2.1	34.2	64.7	122.	186.7	3.
1981	2.2	34.8	66.2	122.	188.2	3.
1982	2.3	35.4	67.7	122.	189.7	3.
1983	2.4	36.0	69.2	122.	191.2	3.

TABLE 4.2

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\gamma(\ell,r,i)$ FOR CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA
(Figures are given in ft/millions ft^3/day)
(NCHu=o reach u acts as a constant head boundary. NCHu=1 reach u acts as a constant flow source)
The sign (-) means that the drawdown at ℓ is not affected by pumpage at r because a constant head boundary is between them.

N C H u		Year	$\gamma(2,r,i)$				$\gamma(4,r,i)$				$\gamma(5,r,i)$				$\gamma(6,r,i)$			
			r				r				r				r			
10	11	i	2	4	5	6	2	4	5	6	2	4	5	6	2	4	5	6
0	0	1	19.6	1.7	--	--	2.0	3.3	--	--	--	--	3.4	1.0	--	--	1.0	11.3
		2	0.1	0.2	--	--	0.3	0.4	--	--	--	--	0.7	0.9	--	--	0.9	3.6
		3	0.	0.	--	--	0.	0.1	--	--	--	--	0.2	0.4	--	--	0.4	1.2
1	0	1	20.5	1.9	0.6	0.2	2.2	4.8	1.3	0.2	0.8	1.3	3.2	1.0	0.2	0.3	1.0	11.5
		2	0.5	0.6	0.5	0.3	0.6	1.7	0.8	0.3	0.5	0.9	0.9	1.0	0.4	0.7	1.0	3.8
		3	0.2	0.2	0.2	0.2	0.2	0.9	0.3	0.2	0.2	0.3	0.3	0.6	0.4	0.5	0.6	1.5
		4	0.1	0.1	0.1	0.1	0.	0.5	0.1	0.1	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.7
0	1	1	9.6	1.7	--	--	2.0	3.3	--	--	--	--	4.6	1.2	--	--	1.3	12.5
		2	0.1	0.2	--	--	0.3	0.4	--	--	--	--	2.1	1.5	--	--	1.6	3.7
		3	0.	0.	--	--	0.	0.1	--	--	--	--	1.	1.	--	--	1.	1.5
		4	0.	0.	--	--	0.	0.	--	--	--	--	0.5	0.6	--	--	0.6	0.7
		5	0.	0.	--	--	0.	0.	--	--	--	--	0.3	0.3	--	--	0.3	0.4
1	1	1	19.3	2.1	0.2	0.	2.1	3.8	0.6	0.	0.9	1.2	5.2	1.4	0.1	0.3	1.4	11.
		2	0.4	0.6	0.5	0.1	0.8	1.2	1.1	0.4	1.0	1.5	3.3	2.1	0.5	0.7	2.1	3.6
		3	0.2	0.2	0.4	0.2	0.4	0.6	1.0	0.6	0.6	1.1	2.2	1.6	0.4	0.7	1.7	1.7
		4	0.	0.1	0.3	0.1	0.2	0.3	0.6	0.5	0.4	0.6	1.5	1.0	0.2	0.5	1.2	0.9

TABLE 4.3

$\beta(k,j,i)$ Values $\left[Ft/Ft^3/Day\right]$ *1000

Wells in Cell 4

Year I	(F-10,J,I)			(F-11,J,I)			(F-16,J,I)		
	J			J			J		
	F-10	F-11	F-16	F-10	F-11	F-16	F-10	F-11	F-16
1	10.00	4.77	2.99	4.82	11.51	4.74	3.05	4.77	9.82
2	0.98	1.04	0.74	1.01	1.32	0.94	0.73	0.95	0.83
3	0.24	0.27	0.19	0.26	0.31	0.23	0.18	0.22	0.17
4	0.07	0.09	0.06	0.08	0.08	0.06	0.05	0.06	0.04
5	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01

TABLE 4.4

The Fairfield Aquifer Area

$\psi_r^u(\ell,n)$ Values $\left[1000\ Ft^3/Day\right]$

(One Unit Pumpage imposed on ℓ during the $i = 1$ Period)

u . . .	10		10		11		12	
r . . .	4		5		5		7	
ℓ . . .	4	2	6	5	4	5	6	5
n								
1	557	220	40	190	60	290	60	10
2	52	120	90	120	130	190	90	20
3	5	20	50	30	80	40	35	10
4	-	-	20	10	30	15	40	5

TABLE 4.5

MAXIMUM INFILTRATION RATES $Q_{INF,\ell}^{u}$ AND STEADY STATE INFILTRATION RATES I_{ℓ}^{u} FROM STREAM REACHES INTO AQUIFER CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA

(Figures are given in acre-ft/day and are based on 325,000 GPD/acre stream bed.)

Reach u / Cell ℓ	10	11	12
4	/ -28.		
5	/ 28.	95. / -20.	
4 & 5	90. / 0.		
7			100. / -3.

LEGEND: $Q_{INF,\ell}^{u}$ / I_{ℓ}^{u}

We can now find out the direct effect of all users' pumping plans on the system response and how this will affect a particular user. The coupling terms, see Section 3.5.2 are determined for the inelastic water use projections at each cell. It is therefore possible to isolate any optimal control problem of any user. The drawdown at each cell due to pumping from all other cells (on the basis of the projected pumping of Table 4.1 is given in Table 4.6. These values are obtained by applying the methodologies as described in Chapters 3 and 4 of Phase II. Table 4.7 summarizes infiltration rates from stream reaches into cells due to the projected imposed pumpage throughout the entire area. Notice that at the end of 1978 stream reaches 10 and 11 (Figure 4.2) are expected to induce maximum infiltration rates into the aquifer. (This last result is already accounted for in the drawdown figures in Table 4.6 after 1978.)

TABLE 4.6

DRAWDOWN AT CELLS IN THE FAIRFIELD-NEW

BALTIMORE AQUIFER DUE TO PUMPING FROM OTHER CELLS

(In Feet)

Year n	Cell ℓ			
	2	4	5	6
1974	2.3	0.1	0.13	2.5
1975	2.6	0.1	0.25	4.9
1976	2.6	0.1	0.30	5.5
1977	2.7	0.1	0.30	5.5
1978	2.8	0.1	0.30	5.6
1979	4.6	1.2	0.5	5.6
1980	4.6	1.2	0.6	5.7
1981	4.9	1.2	0.7	5.7
1982	5.1	1.2	0.7	5.8
1983	5.4	1.2	0.7	5.8

TABLE 4.7

INFILTRATION RATES FROM STREAM REACHES INTO AQUIFER CELLS IN THE FAIRFIELD-NEW BALTIMORE AREA CORRESPONDING TO PUMPAGE PROJECTIONS OVER 10 YEARS

(Figures are given in acre-ft/day)

Year i	$f^{10}(4,i)$	$f^{10}(5,i)$	$f^{11}(5,i)$	$f^{10}(\overline{R},i) = f^{10}(4,i)+f^{10}(5,i)$
1974	-10.9	45.	-10.	34.1
1975	-8.6	46.1	7.8	37.5
1976	-8.0	65.5	46.	57.5
1977	-7.6	88.	70.	80.4
1978	-7.0	97.	95.	90.
1979	-7.0	97.	95.	90.
1980	-7.0	97.	95.	90.
1981	-7.0	97.	95.	90.
1982	-7.0	97.	95.	90.
1983	-7.0	97.	95.	90.

Note that $f^{u}(\ell,i)$ indicates the infiltration in acre-ft/day during period i from the u^{th} stream into the ℓ^{th} cell.

Tables 4.6 and 4.7 provide the terms for decoupling each user's considerations from those of the rest of the users. Table 4.8 lists the aggregated drawdown at each cell over the years resulting from the projected water requirements of all users.

TABLE 4.8

AGGREGATED DRAWDOWN AT CELLS IN THE FAIRFIELD-NEW BALTIMORE AQUIFER OVER TEN YEARS DUE TO AGGREGATED PUMPAGE BY ALL USERS

(In Feet)

Year n	Cell ℓ			
	2	4	5	6
1974	3.6	4.8	9.8	4.5
1975	3.9	5.0	10.	6.9
1976	4.0	5.1	28.	7.5
1977	4.2	5.3	33.	7.5
1978	4.4	5.4	35.	7.6
1979	6.4	5.8	35.	7.7
1980	6.5	7.0	36.	7.8
1981	6.8	6.8	36.5	7.8
1982	7.1	6.9	37.3	7.9
1983	7.5	7.0	38.	7.9

TABLE 4.9

TECHNICAL INFORMATION - WELLS IN THE HAMILTON SOUTH FIELD,

FAIRFIELD-NEW BALTIMORE AREA

Well	Ground Level ft	Steady State Groundwater Level - ft	Depth ft	Maximum Pumpage Capacity acre-ft/day	Initial Lift ft	Maximum Drawdown ft
F-10	581.	548.	200.	13.1	83.	30.
F-11	584.	548.	200.	13.1	86.	30.
F-16	575.	547.	200.	13.1	78.	30.

P(k) cost of pumping 0.0404 $/acre-ft/ft

Currently, the only user who may be concerned with the optimal operation of his wells under the affecting well operations of other users is the City of Hamilton in its South Well Field, Cell 4. It is probably in the interests of other users, in particular the City of Cincinnati, to consider an optimal policy for their water supplies. Nevertheless, the City of Cincinnati Well Field is not yet operating, and in the present state we confine ourselves to the available information, based on the actual situation. In Table 4.9 is some of the model's required technical information for three wells operated by the City of Hamilton in that area. Algebraic technological functions (beta functions) are tabulated in Table 4.3 corresponding to these wells.
A listing of infiltration rates from reach 10 into Cell 4 due to well pumpage inside the cell is given in Table 4.10

TABLE 4.10

$\phi_4^{10}(k,n)$ FLOW BETWEEN STREAM REACH 10 AND CELL 4 AS A FRACTION OF WELL PUMPAGE IN THE HAMILTON SOUTH FIELD, FAIRFIELD-NEW BALTIMORE AREA [(acre-ft/day)/(acre-ft/day)]

Year n	Well		
	F-10	F-11	F-16
1	0.56	0.53	0.60
2	0.06	0.05	0.08
3	0.01	0.01	0.01

The following quadratic mathematical model was solved for the City of Hamilton South Well Field operation:

$$\underset{q(k,n)}{\text{minimize}} \quad \left[Z_4 = \sum_{n=1}^{10} \left\{ (1+r)^{-n} \sum_{k=1}^{3} P(k) \cdot q(k,n) [H(k) \right.\right.$$

$$\left.\left. + \hat{D}(4,n) + \sum_{j=1}^{3} \sum_{i=1}^{n} \beta(k,j,n-i+1) \cdot q(j,i)] \right\} \right]$$

subject to:

$$\sum_{j=1}^{3} \sum_{i=1}^{n} \beta(k,j,n-i+1) \cdot q(j,i) \le D(k)_{max} \qquad n=1,\ldots,10; \quad k=1,2,3$$

$$q(k,n) \le Q(k)_{max} \qquad n=1,\ldots,10; \quad k=1,2,3$$

$$\sum_{k=1}^{3} q(k,n) \ge R(n) \qquad n=1,\ldots,10$$

$$f^{10}(4,10) \le Q_{INF,4}^{10}$$

$$q(k,n) \ge 0 \qquad n=1,\ldots,10; \quad k=1,2,3$$

The various terms in the above formulation are described in detail in section 3.5.1. The control variables $q(1,n)$, $q(2,n)$ and $q(3,n)$

correspond to pumping from wells F-10, F-11, and F-16, respectively, from 1974 - 1983, see Figure 4.1.

Tables 4.1-4.10 provide all necessary information for solving the particular model. The computer program and the solution procedure follow the discussion in section 3.8. Table 4.11 gives the pumping values which minimize the objective function while satisfying the constraints.

TABLE 4.11

OPTIMAL SCHEDULE OF WELL PUMPAGE IN THE HAMILTON SOUTH WELL FIELD, FAIRFIELD-NEW BALTIMORE AREA

Figures are given in acre-ft/day.

Year	Well			Water Requirement
n	1(F-10)	2(F-11)	3(F-16)	R(n)
1974	13.1	13.1	4.4	30.6
1975	13.1	13.1	5.0	31.2
1976	13.1	13.1	5.6	31.8
1977	13.1	13.1	6.2	32.4
1978	6.8	13.1	13.1	33.0
1979	13.1	13.1	7.4	33.6
1980	13.1	8.0	13.1	34.2
1981	8.6	13.1	13.1	34.8
1982	13.1	13.1	9.2	35.4
1983	13.1	13.1	9.8	36.0

Notice that the binding constraints in this particular case are those associated with the maximal capacity of wells. All Lagrange multipliers associated with constraints considering drawdown limits are zero. If the City of Hamilton would like to improve its well operation and reduce operational expenses, it should consider increasing its wells' capacities -- in particular wells F-10 and F-11. A more profound analysis of conclusions which can be drawn by solving such a capacity problem and an example are in the next chapter.

4.3 CONCLUSIONS

This chapter concludes this study's reference to the case study on the Fairfield-New Baltimore area. The following results were achieved by applying the various mathematical developments to this case. A step-by-step illustration of the developing methods and models provided a profound insight into the various functions, procedures and formulations. This chapter constitutes a complete model structure, whereby this study's developments are put together in one structure illustrating the important potential for complex groundwater systems modeling, planning and managing. Once a suitable physical simulation model is available, response functions may be determined. For any set of inputs, these functions provide an explicit computation of the system's time varying response. These functions may thus practically replace the original simulation model. Certainly predictions of water table throughout the aquifer are possible via these functions. Furthermore, these functions allow for the coupling of the system response to pumping with any computational framework such as a management model. The benefit to the Fairfield-New Baltimore area from this study's applications is a by-product which should be studied directly by those who are interested in developing this area's water resources. In particular the M.C.D. has access to both the physical system by means of data acquisition and to the administrative structure by means of the mandate it has to monitor this particular area for reasons described by Speiker, [1968] and Plummer, [1974]. The application of the management model to the studied area restricted the model formulation to the extent that the real present situation defined it. To further illustrate this study's contributions, an imaginary case is considered in the next chapter. This case features most options accounted for in the general model formulation where conjunctive use of ground and surface water are considered.

CHAPTER 5

EXAMPLE PROBLEM

A CONJUNCTIVE USE OF GROUND AND SURFACE WATER SYSTEMS

5.1 INTRODUCTION

In this chapter we formulate a hypothetical system featuring most of what is characterized by the management control model of Chapter 3. The hypothetical system is aimed at showing the prospects of using that model for conjunctive use of ground and surface water systems. In particular are shown the options of water supply from a surface reservoir and artificial recharge from a stream into an aquifer. These options, which are not considered in the previous case study introduce (in addition to the aquifer operation) a new dimension to the problem of water resources optimal control. The physical description (Haimes and Macko, [1973]), requires cooperation among users for effecting drawdowns, and among aquifer, stream and surface reservoir water balance. The goal description requires co-ordination between surface reservoir control and aquifer cells control for the optimum allocation of surface water. The management model objective function as well as the constraints are well adapted to such a problem. The forthcoming discussion should illustrate the applicability and practicability of the model. It shows the variety of conditions under which the model can be successfully utilized, in particular it emphasizes the coupling of a complex groundwater system with a desired management scheme.

5.2 PROBLEM STATEMENT

The problem investigated herein involves a basin comprising aquifers traversed by streams. Water supply systems are assumed to be already developed, consisting of two major elements: pumping wells and surface reservoir. There are L users in the region, to each of whom there corresponds an aquifer cell. The ℓ^{th} user has m_ℓ wells located at the ℓ^{th} cell. There is a single stream traversing all cells. A variable inflow, $\overline{Y}(n)$, enters the basin upstream, and after interacting with aquifer and recharge facilities along its flow, it enters a reservoir of maximal capacity C_m. A surface supply system constructed and operated by a regional agency, pumps water from the reservoir for direct use after proper treatment. Surface water therefore competes with water from wells, and users consider each on a practical economic basis. Finally, each user has the option of transferring water from the stream to the artificial recharge facility in his area so as to recover drawdowns in his aquifer cell.

The problem is formulated and solved on two levels of interactive procedure: The first comprises L optimization programs corresponding to L users in the basin. A particular optimization problem is considered by the ℓ^{th} user for maximizing his net benefit. The gross benefit is due to the quantities of water he consumes over a period of time from both ground and surface water supplies. The costs associated with his water supply are incurred by his using well operations and artificial recharge facilities, and by his consuming quantities of water from the surface water allocated to him. Water use provides him with benefits. For each time period his projected water use activities determine the benefit in dollars per unit of water supply. Technical constraints define the feasible set of decisions the user can make. To execute his optimal policy, the ℓ^{th} user needs information on variables and parameters which are not exclusively under his control. These include drawdown caused by other users, pumping wells, quantities of water available from the stream for artificial recharge, and price and quantities of water available from the surface water system. This information is available on the second level which is comprised of two stages. At the first, the physical system's coupling functions are determined. Resulting from pumping and recharge plans are drawdowns in aquifers and interactions with the stream. The effects of overall activities in the basin on each particular system response can thus be calculated. The second stage of the second level takes care of the surface reservoir operation. An optimization program is carried out. This is aimed at determining the optimal utilization of the surface water supply system. The program is solved subject to reservoir water balance considerations. This balance results from stochastic flow inputs and required outputs of supply. Stage two of the second level provides the first level with the quantities of surface water available for each time period and the associated cost per unit. It is assumed that the cost per unit

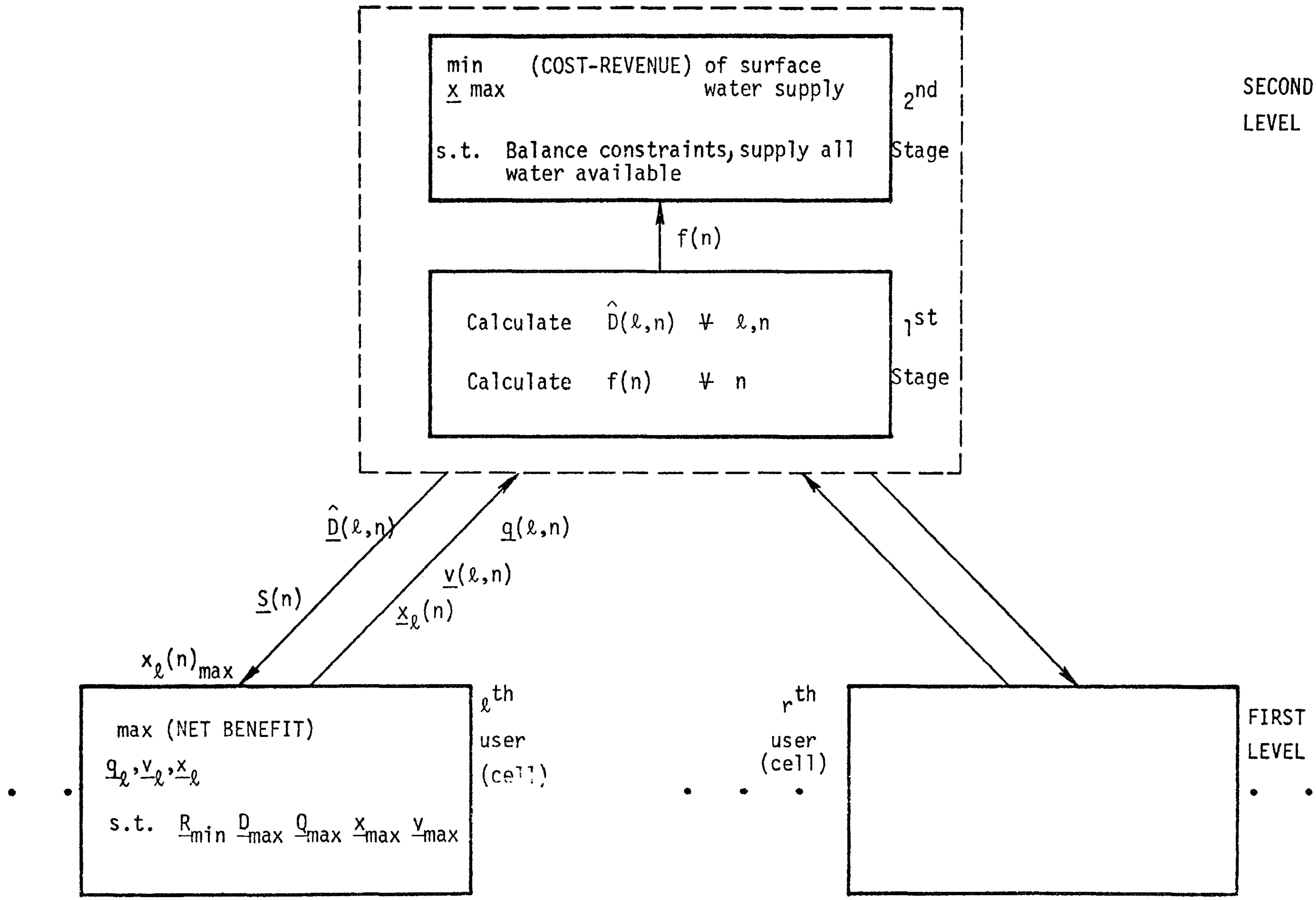


FIGURE 5.1 EXAMPLE PROBLEM MODEL HIERARCHY

of surface water is the same for all users, even if some may affect it more than others. Figure 5.1 shows the model hierarchy. Specific definitions and the different functions involved are discussed further on.

5.3 PROBLEM FORMULATION

5.3.1 First level Optimization Model

Consider the following quadratic model for the ℓ^{th} user:

$$\begin{aligned}
\underset{\underline{q}_\ell,\underline{v}_\ell,\underline{x}_\ell}{\text{minimize}} \quad \Bigg[Z_\ell = \sum_{n=1}^{T} \Bigg[(1+r)^{-n} \Bigg\{ \sum_{k=1}^{m_\ell} P_\ell(k) \cdot q_\ell(k,n) \Big(H_\ell(k) \\
+ \hat{D}(\ell,n) + \sum_{j=1}^{m_\ell} \sum_{i=1}^{n} \beta_\ell(k,j,n-i+1) q_\ell(i,j) \\
- \sum_{i=1}^{n} \gamma(\ell,\ell,n-i+1) v(\ell,i) \Big) + S(n) x(\ell,n) \\
+ V_\ell(n) \cdot v(\ell,n) - W_\ell(n) \Big(\sum_{k=1}^{m_\ell} q_\ell(k,n) \\
+ x(\ell,n) \Big) \Bigg\} \Bigg] \Bigg] \qquad (5.1)
\end{aligned}$$

subject to: $$\sum_{k=1}^{m} q_\ell(k,n) + x(\ell,n) \geq R_\ell(n)_{min}$$

$$n=1,\ldots,T \tag{5.2}$$

$$\hat{D}(\ell,n) - \sum_{i=1}^{n} \gamma(\ell,\ell,n-i+1)v(\ell,i)$$

$$+ \sum_{j=1}^{m_\ell}\sum_{i=1}^{n} \beta_\ell(k,j,n-i+1)q_\ell(j,i) \leq h_\ell(k)_{max}$$

$$n=1,\ldots,T \quad k=1,\ldots,m_\ell \tag{5.3}$$

$$q_\ell(k,n) \leq Q_{max}(k) \qquad n=1,\ldots,T \quad k=1,\ldots,m_\ell \tag{5.4}$$

$$v(\ell,n) \leq v_{\ell max} \qquad n=1,\ldots,T \tag{5.5}$$

$$x(\ell,n) \leq x_\ell(n)_{max} \qquad n=1,\ldots,T \tag{5.6}$$

Where

T is the number of time periods that comprise the planning horizon

r is the interest rate

m_ℓ is the number of wells located at the ℓ^{th} cell

$P_\ell(K)$ is the pumping cost per acre-ft per ft for the k^{th} well

$q_\ell(k,n)$ is the quantity of water pumped from the k^{th} well during the n^{th} period

$H_\ell(k)$ is the lift under steady state conditions at the k^{th} well

$\hat{D}(\ell,n)$ is the drawdown in the ℓ^{th} cell at the end of the n^{th} time period due to aggregate pumpage and recharge in all other cells (by other users) in the region

$\beta_\ell(k,j,n-i+1)$ is the algebraic technological term relating the drawdown at the k^{th} well to the pumping of one unit of water from the j^{th} well during the i^{th} period, and both k and j are located at the ℓ^{th} cell

$\gamma(\ell,r,n-i+1)$ is the algebraic technological term relating the average drawdown at the ℓ^{th} cell to aggregated pumping of one unit of water at the r-th cell during the i^{th} period

$v(\ell,n)$ is the quantity of water used for artificial recharge at the ℓ-th recharge facility during period n

$S(n)$ is the periodical price per acre-ft of surface water supply from the reservoir

$x(\ell,n)$ is the quantity of water supply to the ℓ-th user from the surface reservoir during time period n

$V_\ell(n)$ is the operating cost of recharge per acre-ft in the ℓ-th area with water from the stream

$W_\ell(n)$ is the return per acre-ft of water supply for the ℓ-th user during the n-th period

$R_\ell(n)$ is the minimum water requirements for the ℓ-th user in the n-th period

$h_\ell(k)_{max}$ is the maximum lift allowed at the k-th well due to well design

$Q(k)_{max}$ is the upper limit for pumping from the k^{th} well

$v_{\ell max}$ is the recharge facility capacity limit

$x_\ell(n)_{max}$ is the allocation of surface water supply to the ℓ-th user for the n-th period

The input to the first level from the second level includes

$\hat{D}(\ell,n)$ the drawdown at the ℓ-th cell due to pumpage and recharge in other cells; $S(n)$, the price per unit of water supply from the surface reservoir; $x_\ell(n)_{max}$, the upper limit for quantities of water allocated for the surface water supply. The output from the first level to the second level includes $q_\ell(k,n)$, the pumping plan; $v(\ell,n)$, the artificial recharge plan; and $x(\ell,n)$, the surface water requirement plan.

5.3.2 Second Level - First Stage

Two sets of functions are considered:

$$\hat{D}(\ell,n) = \sum_{\substack{r=1 \\ r=\ell}}^{L} \sum_{i=1}^{n} \gamma(\ell,r,n-i+1) \cdot \left[\sum_{k=1}^{m_r} q_r(k,i) - v(r,i)\right] \qquad (5.7)$$

$\hat{D}(\ell,n)$ is the drawdown observed in the ℓ-th cell area due to the net pumping throughout the rest of the system.

$$f(n) = \sum_{r=1}^{1} \sum_{i=1}^{n} \psi(r,n-i+1) \left[\sum_{k=1}^{m_r} q_r(k,i) - v(r,i)\right] + \sum_{r=1}^{1} I_r \qquad (5.8)$$

where $f(n)$ is the total amount of water induced from the stream into the different aquifer cells during the n-th period.

The values of $\hat{D}(\ell,n)$ are available for updating the first level while $f(n)$ values are used by the second stage of the second level to determine the stream balance.

5.3.3 Second Level-Second Stage

At this stage the operation of the surface reservoir is considered. The following steps are included:

1. Determine the net flow from the stream actually entering the reservoir, $y(n)$:

$$y(n) = \bar{Y}(n) - \sum_{\ell=1}^{L} [f(\ell,n) + v(\ell,n)] - E(n) \qquad (5.9)$$

here $\overline{Y}(n)$ is the stream flow entering the basin upstream, and is naturally a stochastic variable. Assuming variables $Y_1(n), Y_2(n) \ldots Y_M(n)$ with probabilities $p_1, p_2, \ldots, p_M$ then the expected value of $Y(n)$ is $\overline{Y}(n)=E(Y(n))=\sum_{j=1}^{M} p_j Y_j(n)$.

Similar discussion can be found in Buras, [1963].

$f(\ell,n)$ is the quantity of water induced from stream into aquifer in the ℓ-th area, and is determined by the first stage.

$v(\ell,n)$ is the quantity of water from stream transferred into the ℓ-th artificial recharge facility, and results from the first level ℓ-th optimization program.

$E(n)$ is the water loss due to evaporation from stream, reservoir and other facilities, not including overflows due to floods. This quantity, like the upstream flow, is assumed known.

2. Check for the reservoir over-flow. Let C_o and C_m denote the reservoir capacity at the outset of the planning period and the maximum reservoir capacity, respectively. Let

$$\hat{x}(n)=\sum_{\ell=1}^{L} x(\ell,n)$$

$$\left.\begin{array}{l} \text{If } y(n)>C_m-C_o+\sum_{i=1}^{n} \hat{x}(n) - \sum_{i=1}^{n-1} y(i) \\ \text{then} \quad y(n)=C_m-C_o+\sum_{i=1}^{n} \hat{x}(n)-\sum_{i=1}^{n-1} y(i) \end{array}\right\} \quad n=1,\ldots,T \qquad (5.10)$$

3. Consider the cost function for surface reservoir operation: Let the periodic fixed expenses be α_1 \$/period and the operational cost be $\alpha_2\hat{x}(n)+\alpha_3\hat{x}(n)^2$ where $\hat{x}(n)=\sum_{\ell=1}^{L} X(\ell,n)$. The per

unit cost considered for time period n is S(n):

$$S(n) = (\alpha_1 + \alpha_2 \hat{x}(n) + \alpha_3 \hat{x}(n)^2) / \hat{x}(n)$$

The users want the system to provide them with surface water supply while maintaining the most efficient operation. Restrictions are the physical limits and the input-output balance considerations. The agency operating this system does not control the requirements for the water it allocates. It does, however, provide the users with an optimal plan of allocations and the associated cost per unit supplied. A particular plan for surface water allocation is $(\bar{x}(1), \bar{x}(2), \ldots, \bar{x}(T))$, where $\bar{x}(n) = \sum_{\ell=1}^{L} x_\ell(n)_{max}$ is the sum of surface water allocations for all users at period n. Recall, however, that the actual use $x(\ell,n)$ is not necessarily fixed for a given $x_\ell(n)_{max}$, but is limited from above by this allocation, that is $x(\ell,n) \leq x_\ell(n)_{max}$. As a result, $\hat{x}(n) < \bar{x}(n)$ introduces the possibility that an optimal surface water allocation does not necessarily imply full utilization of the available water. Being more realistic, it is possible that some users may wish to consume other users' unused water. Define $\overset{*}{x}(n) = \bar{x}(n) - \hat{x}(n)$ as the amount of water which should be reallocated to these users where the Lagrange multiplier corresponding to the constraint $x(\ell,n) \leq x_\ell(n)_{max}$ is non-zero (meaning that the allocation of surface water $x_\ell(n)_{max}$ is restricting the ℓ^{th} user plans). Overall optimal considerations require that the surplus $\overset{*}{x}(n)$ be allocated according to the values of the associated Lagrange multipliers. But such considerations are not assumed binding in this particular case (each user is interested solely in his own profits). Hence, surplus is shared equally among users who may use it regardless of the marginal benefits. The optimal surface water allocation program is:

$$\underset{\bar{x}}{\text{Minimize}} \sum_{n=1}^{T} (1+r)^{-n} \left\{ (\alpha_1+\alpha_2\bar{x}(n)+\alpha_3\bar{x}^2(n)) - S(n)\bar{x}(n) \right\} \qquad (5.12)$$

Subject to:

1. Quantities available may not exceed the reservoir maximal capacity being also the upper limit for the surface water system supply capacity:

$$\bar{x}(n) \leq C_m \qquad n=1,\ldots,T \qquad (5.13)$$

2. Periodic allocations may not exceed available water in the reservoir:

$$\sum_{i=1}^{n} \bar{x}(i) \leq C_o + \sum_{i=1}^{n} y(i) \qquad n=1,\ldots,T-1 \qquad (5.14)$$

3. Allocations should allow for full utilization of all surface water available over the entire time horizon:

$$\sum_{i=1}^{T} \bar{x}(i) = C_o + \sum_{i=1}^{T} y(i) \qquad (5.15)$$

4. The amount stored in the surface reservoir at the end of each period should not exceed the maximal storage capacity:

$$\sum_{i=1}^{n} \bar{x}(i) \geq \begin{cases} 0 & n: \left\{ \sum_{i=1}^{n} y(i) + C_o < C_m \right. \\ \sum_{i=1}^{n} y(i) + C_o - C_m & \text{Otherwise} \end{cases} \qquad n=1,\ldots,T-1 \tag{5.16}$$

The model formulation in (5.1) - (5.16) is a program for optimal conjunctive use of ground and surface water. It follows the conceptual model represented in Chapter three, with these modifications:

- Construction cost is not considered.
- The penalty cost function for depletion of the stream is originally stated explicitly as a factor in the performance criterion. Here it is given a meaningful application. The surface reservoir operation considers the stream balance. The upper limit $B_\ell(u,n)$, (see Eqn. 3.5), is interpreted through a set of reservoir balance constraints. The penalty term $Q_\ell(u,n)$ is assigned a large value, converting the cost criterion to a set of strict constraints. The infiltration limit constraint in the original model is interpreted here in the second level commonly for all users through the stream balance calculations.

In Figure 5.2 a flow-chart of the model given in (5.1) through (5.16) summarizes the different computations involved.

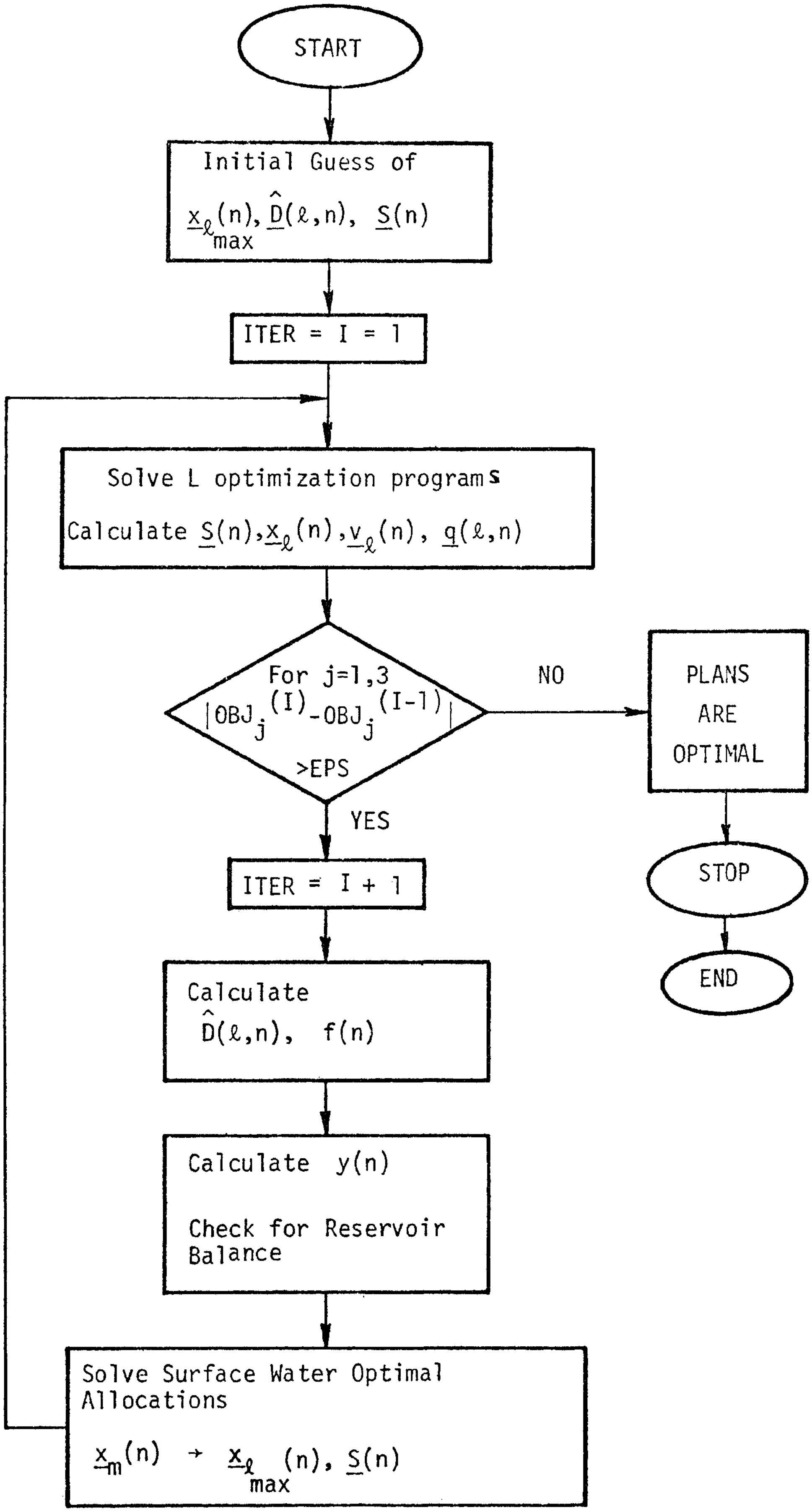


FIGURE 5.2 EXAMPLE PROBLEM PROGRAM FLOW-CHART

5.4 HYPOTHETICAL CASE INPUT DATA AND COMPUTATIONAL RESULTS

In constructing a hypothetical case aimed at illustrating, verifying and refining the model, we believe the data we have generated reflect reality. Realism of information and functions utilized is our main concern. The results obtained from using the generated data and functions are expected to convince the reader as to the model's actuality and prospective applicability.

Three users, L = 3, are in the area. Each operates three wells to meet his water needs, and in addition may choose to buy surface water from the reservoir. Each of two users owns an artificial recharge facility with a limited capacity. The time horizon of planning is six years; application to a longer period is discussed later. Tables 5.1 - 5.7 give the information on the various users. Tables 5.8 - 5.9 show the information applied to the surface reservoir system.

NOTE: The response functions are assumed in effect for three years. Effects of pumpage on the system response after the third year are negligible in this case.

TABLE 5.1

SIX YEARS' PROJECTIONS OF MINIMUM WATER REQUIREMENTS
IN THE HYPOTHETICAL CASE STUDY
(Figures are given in acre-ft/day)

User (cell) / Year n	1	2	3
1	70.	60.	15.
2	70.	65.	15.
3	75.	70.	15.
4	75.	70.	15.
5	75.	70.	15.
6	80.	70.	15.

TABLE 5.2

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\beta(k,j,i)$ FOR WELLS AT EACH OF THE CONSIDERED CELLS IN THE HYPOTHETICAL CASE STUDY

(Figures are given in ft/acre-ft/day)

User (Cell) ℓ	Year i	$\beta_\ell(1,j,i)$ j = 1	2	3	$\beta_\ell(2,j,i)$ j = 1	2	3	$\beta_\ell(3,j,i)$ j = 1	2	3
1	1	.436	.208	.13	.21	.502	.207	.133	.208	.428
	2	.043	.045	.032	.044	.058	.041	.032	.041	.036
	3	.01	.012	.008	.011	.014	.010	.008	.010	.007
2	1	.392	.174	.109	.196	.458	.187	.131	.196	.414
	2	.039	.044	.031	.044	.052	.039	.031	.039	.035
	3	.009	.011	.008	.011	.013	.009	.007	.009	.007
3	1	.349	.153	.006	.183	.436	.179	.122	.183	.392
	2	.037	.041	.030	.039	.048	.037	.028	.037	.033
	3.	.009	.010	.007	.010	.013	.008	.006	.008	.006

TABLE 5.3

ALGEBRAIC TECHNOLOGICAL FUNCTIONS $\gamma(\ell,r,i)$

FOR CELLS IN THE HYPOTHETICAL CASE STUDY

(Figures are given in ft/acre-ft/day)

Year i	$\gamma(1,r,i)$ r 1	2	3	$\gamma(2,r,i)$ r 1	2	3	$\gamma(3,r,i)$ r 1	2	3
1	.044	.009	.004	.009	.039	.007	.002	.013	.035
2	.005	.003	.003	.002	.005	.001	.001	.001	.003
3	.001	.0	.0	.0	.002	.0	.0	.0	.001

TABLE 5.4

$\psi_r(\ell, i)$ FLOW BETWEEN STREAM AND CELLS AS

A FRACTION OF THE PUMPAGE IN THE HYPOTHETICAL CASE STUDY

Year	$\phi_r(1,i)$			$\psi_r(2,i)$			$\psi_r(3,i)$		
	r			r			r		
i	1	2	3	1	2	3	1	2	3
1	.55	.19	.09	.25	.40	.10	.30	.10	.30
2	.05	.12	.01	.01	.10	.01	.05	.01	.10
3	.0	.03	.04	.0	.02	.01	.0	.0	.05

TABLE 5.5

TECHNICAL INFORMATION - WELLS IN THE HYPOTHETICAL CASE STUDY

User (cell) ℓ	Well k_ℓ	Maximum Capacity $Q_\ell(k)_{max}$ [acre-ft/day]	Initial Lift $H_\ell(k)$ [ft]	Maximum Drawdown $D_\ell(k)_{max}$ [ft]	Recharge Facility Maximum Capacity $v_{\ell\ max}$ [acre-ft/day]
	1	20.	70.	25.	
1	2	30.	75.	25.	20.
	3	40.	80.	25.	
	1	30.	100.	25.	
2	2	40.	100.	25.	25.
	3	40.	100.	25.	
	1	7.	150.	20.	
3	2	7.	120.	20.	0.
	3	7.	170.	20.	

P(k) Cost of pumping 0.0404 dollar/acre-ft/ft,

k=1,2,3

TABLE 5.6

EXPECTED BENEFIT PER ACRE-FT OF WATER USE IN THE HYPOTHETICAL CASE STUDY

(In Dollars/acre-ft)

Year i \ User (cell) ℓ	1	2	3
1	54.	56.	60.
2	57.	58.	60.
3	61.	60.	60.
4	64.	62.	60.
5	67.	64.	60.
6	71.	66.	60.

TABLE 5.7

COST OF ARTIFICIAL RECHARGE OPERATIONS
IN THE HYPOTHETICAL CASE STUDY

(In dollars/acre-ft)

User (Cell) ℓ / Year i	1	2	3
1	1.	.7	0.
2	1.	.7	0.
3	1.	.7	0.
4	1.	.7	0.
5	1.	.7	0.
6	1.	.7	0.

TABLE 5.8

EXPECTED VALUES OF FLOWS ENTERING UPSTREAM $\overline{Y}(n)$ AND ANNUAL EVAPORATION RATE $E(n)$ FIGURES FOR THE HYPOTHETICAL CASE STUDY

(In acre-ft/day)

Year n	Upstream Flow $\overline{Y}(n)$	Evaporation Rate $E(n)$	$\overline{Y}(n) - E(n)$
1	300.	80.	220.
2	300.	80.	220.
3	300.	80.	220.
4	300.	80.	220.
5	300.	80.	220.
6	300.	80.	220.

TABLE 5.9

SURFACE RESERVOIR TECHNICAL INFORMATION FOR
THE HYPOTHETICAL CASE STUDY

Initial Reservoir Capacity CAPo = 130.
acre-ft/day

Maximal Reservoir Capacity CAPm = 150.
acre-ft/day

Operation Cost Coefficients:

$\alpha_1 = 20.$ $\alpha_2 = 1.$ $\alpha_3 = .01$

Interest Rate = .08

Figure 5.3 is the optimal solution corresponding to the input data in Tables 5.1 - 5.9. The convergence criterion (Figure 5.2) is $\varepsilon = 100$. The solution is achieved after the fourth iteration. Figure 5.4 represents the solution convergence rate. The computation time on the UNIVAC 1108 digital computer at Case Western Reserve University is 652 seconds and file usage is 114442 words. The solution for the six-year operation period proves that the model constitutes an optimal solution. However, there are at least two difficulties which should be discussed.

First is the difficulty associated with the convergence rate. Two different iterative loops are embedded in the model. One is in the quadratic program subroutine where Wolfe's Algorithm, Wolfe, [1959], is used. This algorithm requires iterative procedure for solving Phase one of the Simplex Tableaux and convergence conditions are well established. The second iterative loop corresponds to the coordination scheme between the two levels (Figure 5.1). It comprises both the physical description and a computational algorithm of transferred parameters and functions between the two levels. The resulting procedure is actually not related to any known coordinating algorithm (Haimes and Macko, [1973], Lasdon, [1970]). The coordination is merely an information flow among users and between them and the surface water supply system. Each user sets his own policy, but there is no overall regional management policy. We could not find any analytical approach by which to prove conditions for convergence. We can only say that all ten different runs of the program utilizing different input data showed consistency with regard to the convergence rate. No run iteration number exceeded 5. The other difficulty is the dimensionality of the program. In particular the planning horizon plays a critical role in the program's size. A one-unit increase in the planning period introduces to each program at the first level 3 + k decision variables, where k is the number of wells associated with a particular user. The number of constraints is increased by 4 + 2k. In the second level it adds two decision variables and four constraints to the surface reservoir optimization model. Figure 5.5 is a graph of the computation time versus the planning time for this case study.

We conclude this discussion by stating that the model is available for use and is capable of solving larger-sized problems. Of course, the trade-off between computation time and computer capacity should be considered.

To complete this model analysis, a sensitivity analysis was carried out. It should provide some guidance for any future developments based on this model, in particular with respect to information and data acquisition.

User	Year	Well Pumping Plan 1	2	3	Recharge Plan	Surface Water Use Plan
1	1	15,038	30,000	40,000	20,000	53,333
	2	20,000	13,462	40,000	20,000	30,278
	3	20,000	12,897	40,000	20,000	15,216
	4	20,000	12,864	40,000	20,000	14,444
	5	16,040	30,000	15,146	20,000	18,649
	6	20,000	30,000	12,122	20,000	18,789
2	1	4,456	40,000	40,000	25,000	53,333
	2	,000	40,000	19,921	25,000	30,278
	3	30,000	12,382	40,000	25,000	15,216
	4	30,000	10,176	40,000	25,000	14,444
	5	30,000	9,655	40,000	25,000	18,649
	6	30,000	9,769	40,000	25,000	18,789
3	1	7,000	7,000	7,000		43,333
	2	7,000	7,000	7,000		30,278
	3	7,000	7,000	7,000		15,216
	4	7,000	7,000	7,000		14,444
	5	7,000	7,000	7,000		18,649
	6	7,000	7,000	7,000		18,789

Surface Water Per Unit Cost

Year	1	2	3	4	5	6
$/acre-ft	2.616	2.105	1.854	1.852	1.883	1.885

FIGURE 5.3. EXAMPLE PROBLEM - THE SIX-YEAR OPTIMAL SOLUTION

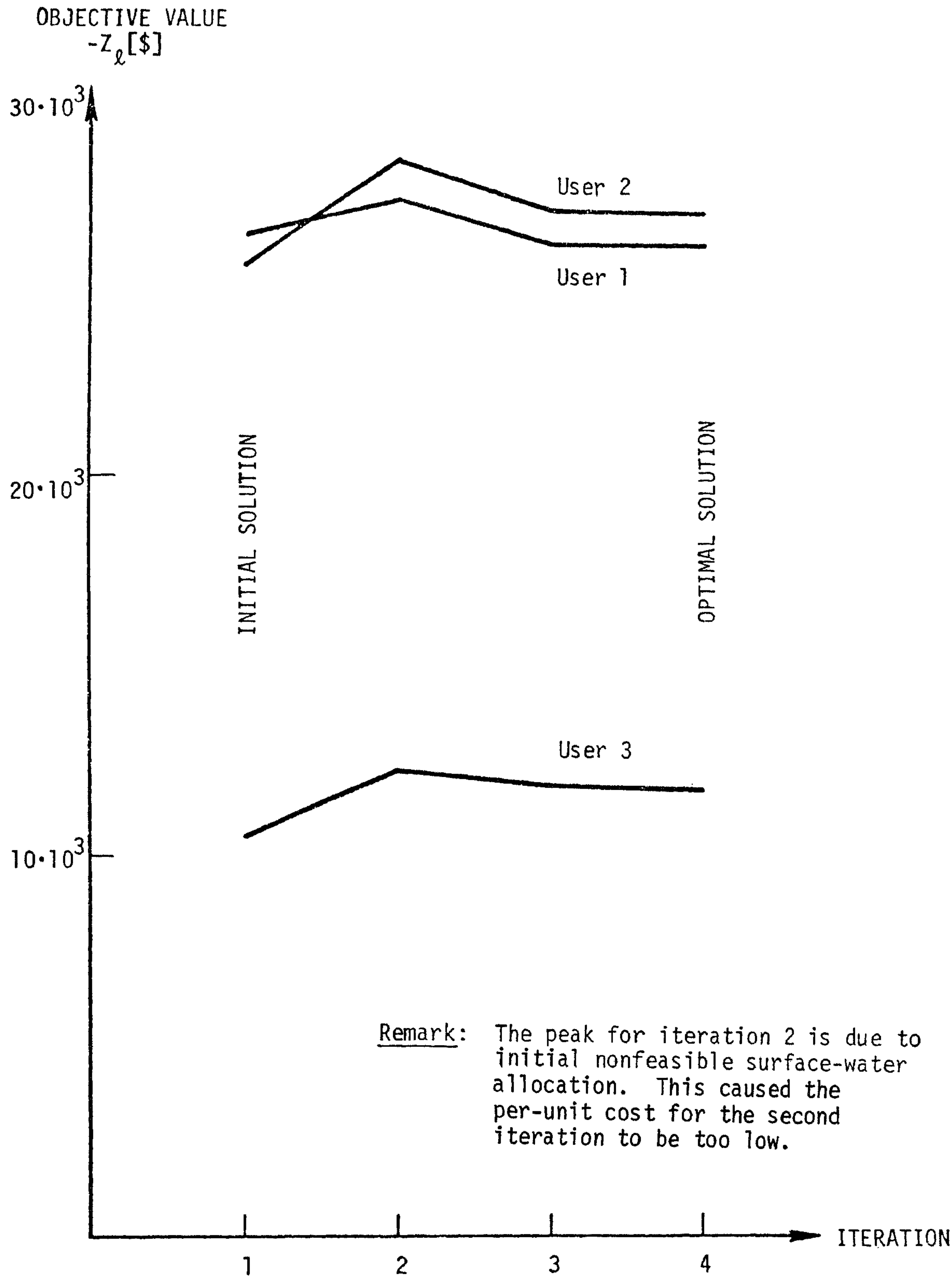


FIGURE 5.4. CONVERGENCE RATE OF OPTIMAL SOLUTION. CASE II, SIX-YEAR OPERATION.

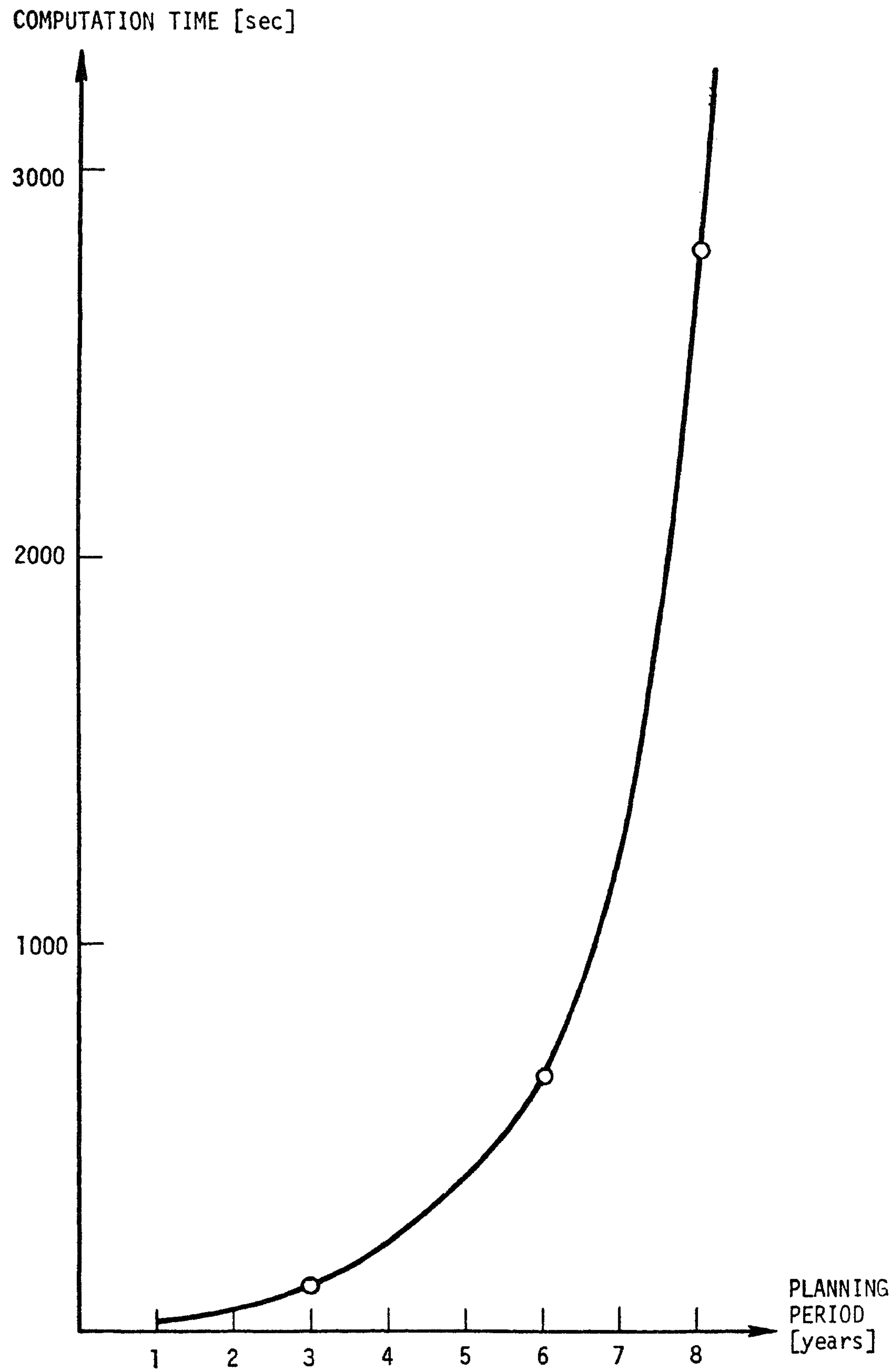


FIGURE 5.5. COMPUTATION TIME VERSUS PLANNING PERIOD, EXAMPLE PROBLEM, UNIVAC 1108.

5.5 SENSITIVITY ANALYSIS

The main purpose of the forthcoming discussion is to point out some elements of concern associated with this model. A sensitivity analysis of different aspects in the model should assist in that task. To save computer time, the sensitivity analysis was performed for a three-year planning period.

5.5.1 The objective's value and the upstream flow

Particular care should be given to the input data. This is especially true because probabilistic data introduce uncertainty into the basic results. Figure 5.6 represents the sensitivity of the optimal solutions by means of the objective value to the probabilistic stream flow. It is clear that each user's performance is linearly dependent on the net upstream flow. This flow is essentially the measure of surface water availability. An interesting comparison is made in Table 5.10 where the slopes of the curves in Figure 5.6 are compared with the Lagrange multipliers associated with the constraints (5.6). These constraints limit the available surface water for each time period. The multipliers are interpreted as the cost per unit of upstream flow. Its relation to actual operational plans is discussed in Section 5.5.2.

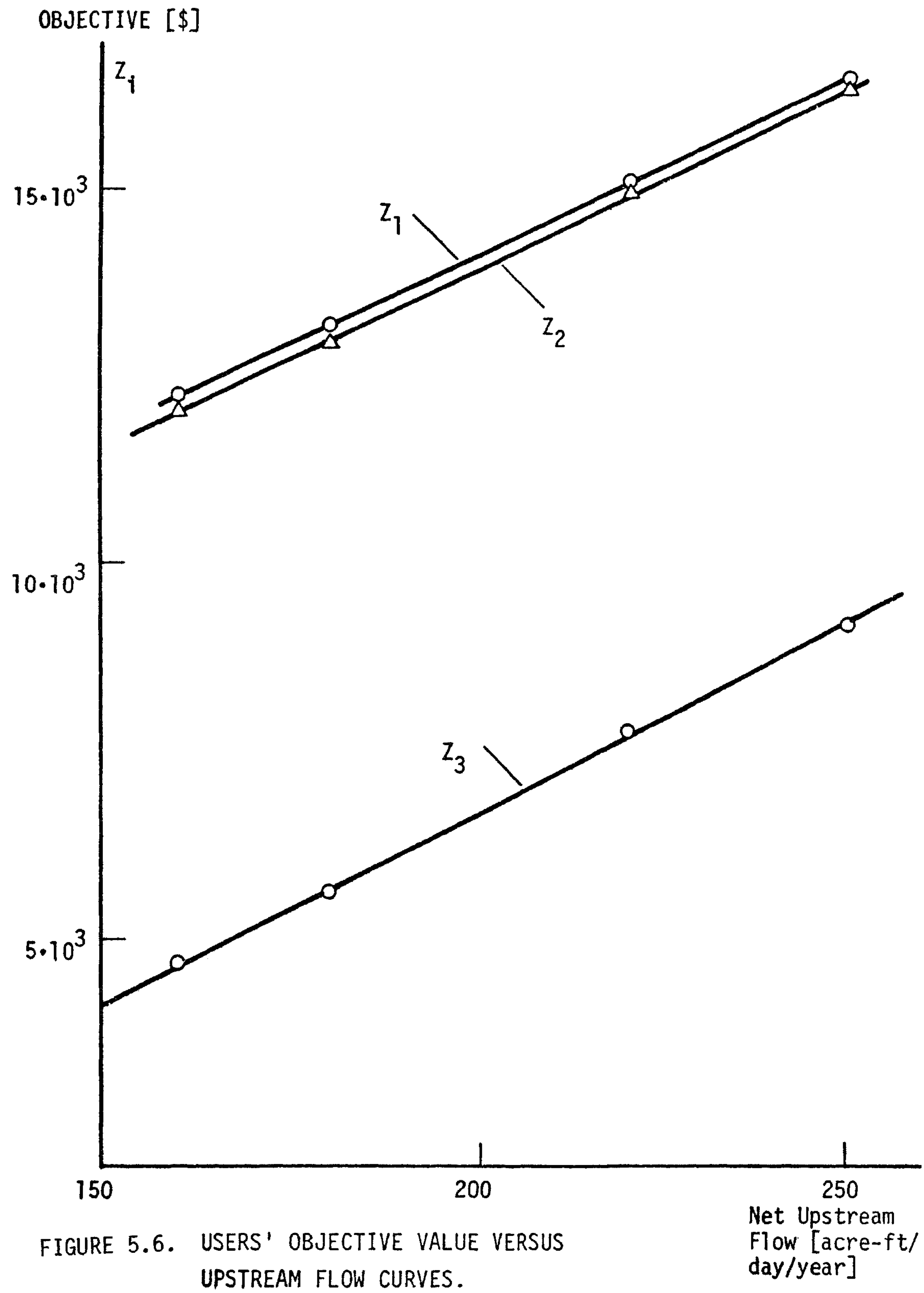


FIGURE 5.6. USERS' OBJECTIVE VALUE VERSUS UPSTREAM FLOW CURVES.

TABLE 5.10

A COMPARISON BETWEEN THE SLOPES OF THE OBJECTIVE VERSUS STREAM FLOW CURVES AND THE LAGRANGIANS ASSOCIATED WITH SURFACE WATER AVAILABILITY CONSTRAINTS

User	Year i	Lagrange Multiplier λ_i	$\bar{\lambda} = (\sum_{i=1}^{3} \lambda_i)/3$	Slope of Sensitivity Curve
1	1	47.5	47.2	47.
	2	47.0		
	3	46.9		
2	1	49.4	47.8	48.
	2	47.8		
	3	46.1		
3	1	53.1	49.2	50.
	2	49.5		
	3	46.1		

5.5.2 The Operational Plans and the Upstream Inflow

The information generated for the hypothetical case assigns high priorities to water use. This should spur the optimal operation planners to utilize all available water sources. Hence, a decrease in one source such as surface water availability should not affect pumping plans. However, it will affect the surface water use plans. This effect is illustrated in Figure 5.7. The probabilistic nature of stream flow in this case is eventually a factor in considering surface water use. Another component which is dependent upon stream flow is artificial recharge. This activity certainly competes with surface water supply for quantities from the stream. In this model each user's independent policy causes him to disregard any possible benefit to him from having more surface water to use if he uses less water for artificial recharge. The various users could realize immediate benefits if they would coordinate their artificial recharge activities.

5.5.3 The Effect of Aggregated Drawdown

A particular user's program considers the drawdown caused by other users (the term $\hat{D}(\ell,n)$ both in the objective cost function (5.1) and in the upper limit for drawdown constraint (5.3). The sensitivity of the objective value to changes in $\hat{D}(\ell,n)$ is well defined by the Lagrange multipliers associated with the constraints (5.3). In Table 5.11 are the corresponding multipliers' values for the three users' optimal plans. These are interpreted as the dollar value of a unit drawdown.

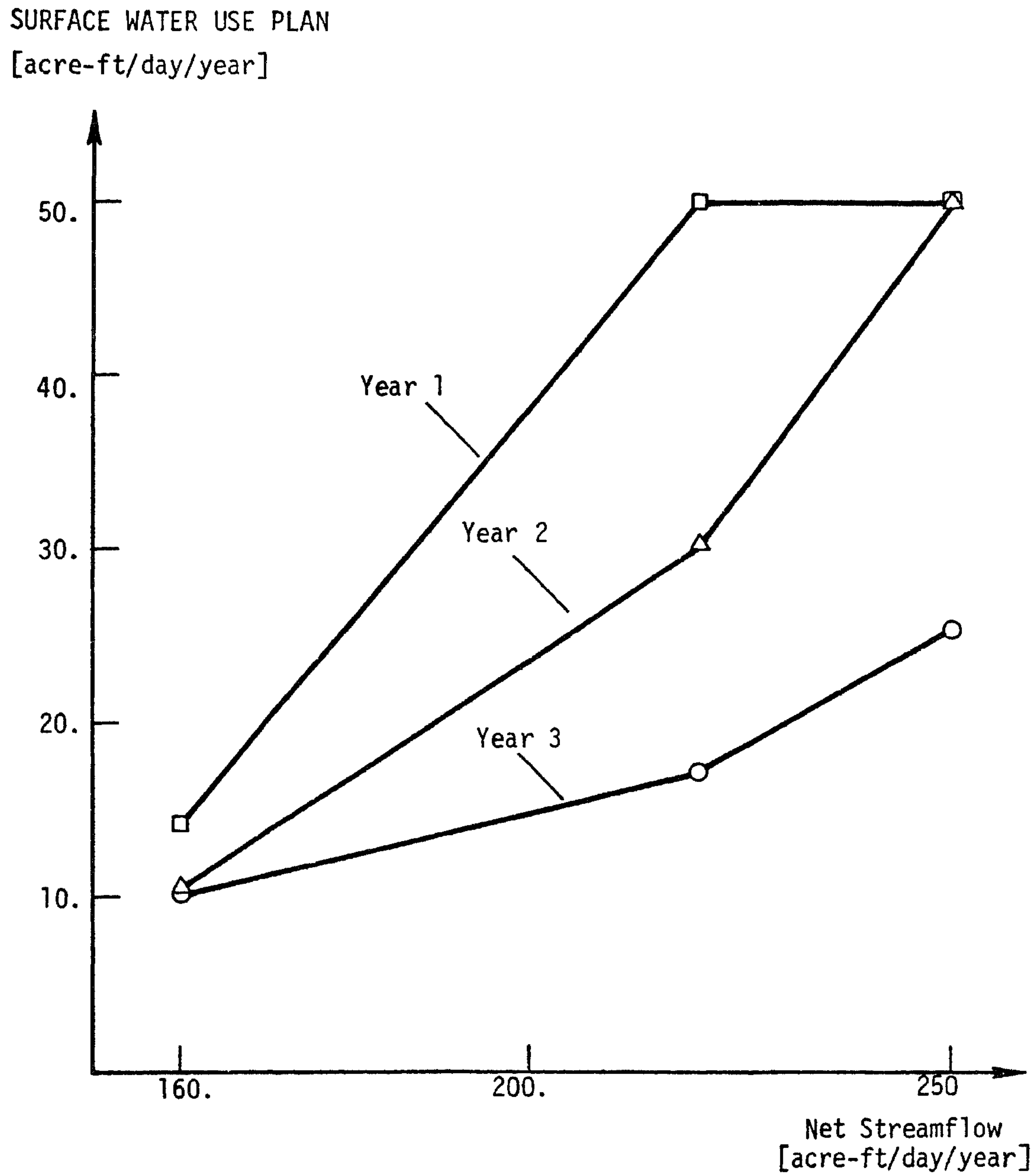


FIGURE 5.7. SURFACE WATER PLAN VERSUS UPSTREAM FLOW.

TABLE 5.11

LAGRANGE MULTIPLIERS ASSOCIATED WITH LIMITING DRAWDOWN CONSTRAINTS UNDER AN OPTIMAL OPERATION PLAN

Year	User (cell) 1		User (cell) 2		User (cell) 3	
i	Well k	$\lambda_1(k,i)$ \$/ft	Well k	$\lambda_2(k,i)$	Well k	$\lambda_3(k,i)$
	1	0.	1	0.	1	0.
1	2	75.8	2	85.1	2	0.
	3	0.	3	0.	3	0.
	1	0.	1	0.	1	0.
2	2	47.8	2	52.9	2	0.
	3	69.3	3	72.8	3	0.
	1	0.	1	0.	1	0.
3	2	57.9	2	62.1	2	0.
	3	76.6	3	76.6	3	0.

TABLE 5.12

THE OPERATIONAL PLANS AND PERTURBATIONS IN THE UPPER LIMIT FOR DRAWDOWN

User (Cell)	Year i	Well k	$D_2(2)_{max}$ = 25. ft		$D_2(2)_{max}$ = 24. ft	
			Surface Water Use [acre-ft/day/year]	Well Pumpage [acre-ft/day/year]	Surface Water Use [acre-ft/day/year]	Well Pumpage [acre-ft/day/year]
1	1	1	50.	20.	50.	20.
		2		30.		30.
		3		34.96		34.74
	2	1	30.26	20.	32.96	20.
		2		13.58		13.36
		3		40.		40.
	3	1	15.22	20.	15.42	20.
		2		12.87		12.88
		3		40.		40.
2	1	1	50.	4.45	50.	0.
		2		40.		40.
		3		40.		39.32
	2	1	30.26	0.	32.96	0.
		2		40.		40.
		3		19.93		15.76
	3	1	15.22	30.	15.42	30.
		2		12.38		13.29
		3		40.		40.

The effect of perturbations in $\hat{D}(\ell,n)$ on the operation plans is much more complicated and may not be explicitly derived from the optimal solution information. These perturbations are more significant in affecting the drawdown constraints (5.3). (The effect of these in the objective function can be measured by conducting a sensitivity analysis on the initial lift, $H_\ell(k)$. This is found to have no effect on the operational plan.)

Perturbations in $\hat{D}(\ell,n)$ with respect to the constraints (5.3) are essentially equivalent to perturbations in the upper limit for drawdown $D_\ell(k)_{max}$. In Table 5.12 a sensitivity analysis of the operational plan to limit drawdown is summarized. A unit change in $D_2(2)_{max}$ for the second user is introduced. Eventually the operational plan is unpredictably sensitive to such perturbations.

5.6 EXAMPLE PROBLEM SUMMARY AND CONCLUSIONS

Applying the management model developed in this study to the hypothetical case achieved these goals:

1. A full utilization of the model for a realistic hypothetical case.
2. A step-by-step analysis of conjunctive use of ground and surface water systems.
3. A profound analysis of advantages, drawbacks and prospective uses of the proposed model formulation, solution and implementation.

The Example problem analysis completes the development of the management control model analyzed in Chapter 3. It illustrates the potential inherent in the model for an even more detailed analysis of the conjunctive use of ground and surface water. A solution to the problem for short-term operation planning is given and is proved to be stable and satisfactory.

The trade-off between computer time and capacity should be further studied. The model results in optimal operational plans for water use and the associated value of the performance criterion. It illustrates one more time how the response functions can actually be used to couple a groundwater system model with a large-scale management model. The sensitivity analysis points out that if the performance function depends on input information, changes in the objective value caused by input variations can be predicted and evaluated once a given deterministic input is solved. On the other hand, optimal operative plans are heavily dependent upon some of the model's parameters in an unpredictable way. It is therefore necessary to first identify physical parameters of the system as accurately as possible. These include transmissivity and storage coefficients upon which the algebraic technological functions are dependent. Also the physical coefficients related to the stream bed are needed for accurate estimation of infiltration rates. The stream flow probabilistic features are important if surface water is the main source of supply for regional development. This model is a possible tool for evaluation of this factor and for possible compensation of groundwater supply in case surface water is lacking.

The discussion in the forthcoming chapter is directed toward the analysis of a different type of problem associated with water resources system management. It is the problem of multiobjective functions faced in regional management by the systems analyst. In particular are considered economic, environmental and energy conservation measures. Some of this study's developments are embedded in the structuring of the model, particularly in the objective formulation.

CHAPTER 6

APPLICATION OF MULTIOBJECTIVE ANALYSIS (THE S.W.T. METHOD) TO A CASE STUDY

6.1 INTRODUCTION

The definition and formulation of real world problems in several systems involve problems with two or more noncommensurable objectives. The formulation of methods for the analysis and optimization of multiple objective functions shared a common goal of avoiding the large computational effort associated with these problems [see Haimes and Hall, 1974, and Haimes, Hall and Freedman, 1975], [Cohon, 1974].

The Surrogate Worth Trade-off Method has been chosen to solve a problem in water resources systems. The area is the Fairfield-New Baltimore area of the Miami Conservance District. A description of hydrological, geological and water use is found in Phases I and II of the earlier study, [Haimes, 1973, 1974].

The system consists of an unconfined, well-defined aquifer hydraulically connected with the Great Miami River which traverses the aquifer. Pumping is developed in four cell fields. The water supply is expected to satisfy the needs of municipalities and industries in the area.

The coupling of a complex multicell aquifer system is described by the algebraic technological functions for cells. The objective functions depend on the algebraic technological functions and include functions developed to relate the infiltration from a stream into an aquifer due to the pumping at the cells. For more details see the discussion in previous chapters.

6.2 PROBLEM DEFINITION

The problem is that of determining the maximum allowances for developing groundwater and surface water supply in the Fairfield area in order to optimize the following non-commensurable multi-objective functions:

- Maximize benefits with respect to water use.
- Minimize the cost of pumping.
- Minimize the cost associated with water supply from the stream.
- Maximize environmental regional benefits, by minimizing the amount of water taken from the stream by means of infiltration and water supply.
- Minimize energy consumption due to groundwater pumpage and surface water supply.

The first of three non-commensurable objectives listed above represents the economical objective, for which there is an actual equation that defines it. The second objective function, environmental considerations, can be obtained from the regional authorities. In the next section of this chapter there is a complete explanation of this objective. Basically, the third objective function, energy consumption, was developed by us.

The development of surface water supply has been included in this formulation. In each objective function the terms corresponding to the surface water are considered. Hence, the original objective functions are altered by the term corresponding due to the consideration of surface water supply.

6.3 PROBLEM FORMULATION

Solve the non-commensurable multiobjective function problem:

$$\min \{\hat{f}_1(\underline{Q}),\ f_2(\underline{Q}),\ f_3(\underline{Q})\}$$
$$\text{subject to}$$
$$g_k(\underline{Q}) \leq 0 \qquad k = 1,2,\ldots,m \tag{6.1}$$

The definition of the objective functions and variables are:

$\hat{f}_1(Q)$ = economical objective function.

$f_2(\underline{Q})$ = environmental objective function.

$f_3(\underline{Q})$ = energy consumption objective function.

$g_k(\underline{Q})$ = system constraints.

Definition of parameters,

$Q(\ell,n)$ = amount of pumping water from the ℓ^{th} cell during the nth time period.

$S(\ell,n)$ = an averaged drawdown at the ℓth cell at the end of the nth time period due to the amount of pumping water.

$B(n)$ = the benefit to the user per unit of pumping water and stream water supply at the nth time period.

r = discount rate.

$C_o(\ell)$ = cost per unit of flow per unit of lift of water at the ℓth cell.

$H(\ell)$ = the lift under steady state conditions at the ℓth cell.

$C_x(n)$ = the cost of treatment per unit of water to be supplied from the stream at the nth time period.

$f(n)$ = quantity of water induced from the stream into the cells during the nth time period (infiltration).

$X(n)$ = amount of water taken from the stream to be supplied at the nth time period.

$\gamma(\ell,j,n-i+1)$ = an averaged drawdown at ℓth cell in the nth time period due to one unit of pumping water in the jth cell in the ith period.

$\psi(\ell,n-i+1)$ = quantity of water induced from the stream into the cells at the end of the nth time period due to one unit of pumping water at the ℓth cell during the ith time period.

$V(n)$ = energy consumption per unit of water taken from the stream at the nth time period.

$R(n)$ = projected demand of water at the end of the nth time period.

The averaged drawdown equation and the infiltration equation has been developed in Phase II. These equations have the following form:

$$S(\ell,n) = \sum_{j=1}^{R} \sum_{i=1}^{n} \gamma(\ell,j,n-i+1)Q(j,i) \tag{6.2}$$

and,

$$f(n) = \sum_{\ell=1}^{R} \sum_{i=1}^{n} \psi(\ell,n-i+1)\ Q(\ell,i) \tag{6.3}$$

In the summations, R represents the number of cells.

The present problem formulation includes groundwater supply and surface water supply as follows.

The first objective function (economic) is defined as follows:

$$\max_{\underline{Q},\underline{X}} f_1(\underline{Q},\underline{X}) = \sum_{n=1}^{T} \{B(n)[[\sum_{\ell=1}^{R} Q(\ell,n)] + X(n)]/(1+r)^n\}$$

$$- \sum_{n=1}^{T} \sum_{\ell=1}^{R} \frac{C_o(\ell)}{(1+r)^n} \cdot [H(\ell) + S(\ell,n)] \cdot Q(\ell,n)$$

$$- \sum_{n=1}^{T} \frac{C_x(n) \qquad X(n)}{(1+r)^n} \tag{6.4}$$

The first term represents the benefits from the water use; where B(n) can be a constant value related to n, or a function of the quantity of water. The second term represents the quadratic cost of pumping water. The third term represents the cost of treatment of the water to be supplied from the stream.

The second objective function (environmental) is defined as follows:

$$\min_{\underline{Q},\underline{X}} \; f_2(\underline{Q},\underline{X}) = \sum_{n=1}^{T} \{ [\sum_{i=1}^{n} \sum_{\ell=1}^{R} \psi(\ell,n-i+1)Q(\ell,i)] + X(n)\} \tag{6.5}$$

The first term in the second objective function represents the quantity of water induced from the stream into the cells. The second term represents the water taken from the stream for direct supply. This objective is a linear objective function. We assume that this objective has been given by the regional authorities.

The regional authorities would like to minimize the risk of the stream going dry, and at the same time, minimize the possibilities that the pollution of the stream will be spread over the Fairfield-New Baltimore.

The third objective function (energy consumption) is defined as follows:

$$\min_{\underline{Q},\underline{X}} \; f_3(\underline{Q},\underline{X}) = \sum_{n=1}^{T} \sum_{\ell=1}^{R} P(n)[H(\ell) + S(\ell,n)]\, Q(\ell,n)$$
$$+ \sum_{n=1}^{T} P(n)\; V(n)\; X(n) \tag{6.6}$$

The first term in this objective function represents the energy consumption used by the aggregate pumping in all the cells. The second term represents the energy consumption used for the stream water supply.

P(n) is the penalty imposed on planning for energy use in the future. It is expected to increase with time due to uncertainty of energy availability and prices. This variable to be defined can follow a complete study by the experts on planning for energy use.

The system constraints, $g_k(\underline{Q},\underline{X})$, constitute a set of restrictions on the decision variables explicitly and implicitly.

- Minimum water requirements must be met

$$\sum_{\ell=1}^{R} Q(\ell,n) + X(n) \geq R(n) \qquad n=1,2,\ldots,T \tag{6.7}$$

with this constraint, the water demands of the municipalities and industries are satisfied.

- Pumping capacity must not be exceeded

$$Q(\ell,n) \leq Q_{max}(\ell) \qquad \begin{matrix} n=1,2,\ldots,T \\ \ell=1,2,\ldots,R \end{matrix} \tag{6.8}$$

- Surface water supply system capacity must not be exceeded

$$X(n) \leq X_{max}(n) \qquad n-1,2,\ldots,T \tag{6.9}$$

the last two constraints satisfy the construction and technical limitations.

- Water Balance in the stream must be maintained

$$f(n) + X(n) \leq WB_{max}(n) \quad n=1,2,\ldots T \tag{6.10}$$

this constraint keeps the stream from going dry, satisfying the environmental objectives.

These parameters are defined as follows:

$Q_{max}(\ell)$ = maximum capacity of pumping at the ℓth cell.

$X_{max}(n)$ = maximum capacity of stream water supply system at the nth time period.

$WB_{max}(n)$ = maximum quantity of water that can be taken from the stream at the nth time period.

The regional authorities who have a well-defined idea of the limitations and range of the objective functions and decision variables can be the decision makers to solve this multiple-objective problem. To use the Surrogate Worth Trade-off Method in a minimization form it will be necessary to arrange the objectives thusly: First, the economic objective has been given as a maximization problem. In order to change this to a minimization problem, the following will be used.

$$\max f_1 = -\min(-f_1) \tag{6.11}$$

then the minimization objective function is:

$$\hat{f}_1 = -f_1 \quad .$$

Therefore, the economical objective function is:

$$\min_{\underline{Q},\underline{X}} \hat{f}_1 = - \sum_{n=1}^{T} \{B(n)[[\sum_{\ell=1}^{R} Q(\ell,n)] + X(n)]/(1+r)^n\}$$

$$+ \sum_{n=1}^{T} \sum_{\ell=1}^{R} \frac{C_o(\ell)}{(1+r)^n} \cdot [H(\ell) + S(\ell,n)] \cdot Q(\ell,n)$$

$$+ \sum_{n=1}^{T} C(n)\ X(n)/(1+r)^n \tag{6.12}$$

which is essential to minimize the net cost of the water supply.

Second, the environmental objective function will remain exactly as it was formulated above:

$$\min_{\underline{Q},\underline{X}} \; f_2 = \overset{T}{\underset{n=1}{}} \; [f(n) + X(n)]$$

$$= \sum_{n=1}^{T} \{ [\sum_{\ell=1}^{R} \sum_{i=1}^{n} \psi(\ell, n-i+1) Q(\ell, i)] + X(n) \qquad (6.13)$$

This objective function is given by the regional authorities; its range is limited by the infiltration and stream water supply constraints.

The third objective remains the same:

$$\min_{\underline{Q},\underline{X}} \; f_3 = \sum_{n=1}^{T} \sum_{\ell=1}^{R} P(n)[H(\ell) + S(\ell,n)] \cdot Q(\ell,n) + \sum_{n=1}^{T} P(n)V(n)X(n) \qquad (6.14)$$

The set of constraints will define the space . Hence. $\underline{Q},\underline{X} \in \Omega$.

6.4 PROBLEM APPLICATION

The Fairfield-New Baltimore aquifer and stream system in the Great Miami River valley is to be studied. This system incorporates four cells into the multicell aquifer groundwater system, and a stream traversing the aquifer. The problem is to find the optimum amounts of water to be supplied for each of the cells and the stream to meet the needs of the industries and domestic users of the area.

The necessary data are available in Phase II and Chapter 4 of this phase. Also extra data were generated.

The data utilized are:

Table 4.2:	Algebraic Technological Functions.	CHAPTER 4
Table 4.4:	Water induced or infiltration.	
Table 6.1:	Water requirements projections.	

Additional data:

$Q_{max}(\ell)$	= 13 acre ft/day	$\ell=1,\ldots,4$
$WB_{max}(n)$	= 35 acre ft/day	$n=1,2,3$
$X_{max}(n)$	= 20 acre ft/day	$n=1,2,3$
$B(n)$	= 10.0$/acre ft.	$n=1,\ldots 3$
$C_o(\ell)$	= 0.05 $/acre ft.	$\ell=1,\ldots,4$
$H(\ell)$	= 80 ft.	$\ell=1,\ldots,4$
r	= 0.08	

TABLE 6.1

Water requirements projections in the Fairfield-New Baltimore area. Figures are given in acre ft/day.

Year i	Demand R(i)
1	30
2	31
3	32

$C_x(n)$ = 6.0 \$/acre ft

$V(n)$ = 20 ft. acre ft/day   n=1,2,3

P(n)

Penalty imposed on planning for energy use

n	1	2	3
P(n)	.1	.12	.14

In this problem there are data available to do the corresponding study over a period of three years.

The economic objective function will be chosen as the one that dominates all other objective functions. Therefore, the optimization can follow classical forms, where the secondary objectives can be handled as constraints. At this point, the ε-constraint approach will be used.

Analyzing the form of the objective functions, the first and third objective functions are quadratic, the second is linear. The GRG is a FORTRAN code which solves this kind of problem.

In order to facilitate the study, the Fairfield-New Baltimore area has been divided into different regions where the cells and the stream are shown. In this study cells 2,4,5 and 6, and the stream are utilized, (see Figure 4.2).

The first step of the Surrogate Worth Trade-Off method is to find the minimum levels for each objective function. To solve the objective functions the GRG computer program available at the UNIVAC 1108 has been used.

To find the minimum values for each objective function, solve

$$\min_{\underline{Q},\underline{X}} f_2$$

subject to $\underline{Q}$ and $\underline{X} \varepsilon \Omega$

by using the GRG computer program that solves non-linear problems, the solution is:

$$\min f_2 = 15.527 = \varepsilon_{2min}$$

where the decision vector is

	Q(ℓ,n)				X(n)
	ℓ				
n	2	4	5	6	
1	13.0	13.0	4.0	0.0	0.0
2	13.0	13.0	0.0	5.0	0.0
3	13.0	0.0	6.0	13.0	0.0

To find the minimum objective value of the third objective, solve:

$$\min_{\underline{Q},\underline{X}} f_3$$

subject to $\underline{Q}$ and $\underline{X} \varepsilon \Omega$

the solution is

$$\min f_3 = 581.656 = \varepsilon_{3min}$$

where the decision vector is

	Q(ℓ,n)				X(n)
	ℓ				
n	2	4	5	6	
1	0.0	5.28	4.72	0.0	20.0
2	0.49	4.88	4.85	0.78	20.0
3	0.51	5.29	5.23	0.97	20.0

The decision variables have lower and upper bounds, and besides, there is a constraint in the infiltration. Hence, the maximum level of the second and third objectives was found by using the GRG computer program again.

To find the maximum level of the second objective function, solve:

$\max f_2 = 87.615 = \varepsilon_{2max}$

the decision variables are:

	Q(ℓ,n)				X(n)
n	ℓ				
	2	4	5	6	
1	0.0	0.0	5.61	13.0	20.0
2	13.0	0.0	13.0	13.0	20.0
3	13.0	13.0	13.0	13.0	6.82

To find the maximum level of the third objective function, solve:

$\max f_3 = 3351.58 = \varepsilon_{3max}$

the decision variables are:

	Q(ℓ,n)				X(n)
n	ℓ				
	2	4	5	6	
1	13.0	13.0	13.0	13.0	17.35
2	13.0	13.0	13.0	13.0	8.23
3	13.0	13.0	13.0	13.0	5.30

Rewrite the problem using the ε-constraint approach:

$$\min_{\underset{\sim}{Q},\underline{X}} \hat{f}_1 = -10 \left\{ \sum_{n=1}^{3} \sum_{\ell=1}^{4} [Q(\ell,n)+X(n)]/(1.08)^n \right\}$$

$$+ \sum_{n=1}^{3} \sum_{\ell=1}^{4} \frac{0.05}{1.08^n} \quad [80+S(\ell,n)]Q(\ell,n)+6 \sum_{n=1}^{3} X(n)/(1.08)^n \tag{6.15}$$

subject to

$$\sum_{n=1}^{3} \left[\sum_{\ell=1}^{4} \sum_{i=1}^{n} \psi(\ell,n-i+1)Q(\ell,i) + X(n) \right] \leq \varepsilon_2 \tag{6.16}$$

$$\sum_{n=1}^{3} \sum_{\ell=1}^{4} P(n)[80+S(\ell,n)]Q(\ell,n) + 20 \sum_{n=1}^{3} P(n)X(n) \leq \varepsilon_3 \tag{6.17}$$

$$R(n) - X(n) - \sum_{\ell=1}^{4} Q(\ell,n) \leq 0 \qquad n=1,2,3 \tag{6.18}$$

$$\sum_{\ell=1}^{4} \sum_{i=1}^{n} \psi(\ell,n-i+1) + X(n) - 35 \leq 0 \quad n=1,2,3 \tag{6.19}$$

$$\begin{aligned} &Q(\ell,n) \geq 0 && \ell=1,\ldots,4 \; ; \; n=1,2,3 \\ &X(n) \geq 0 && n=1,2,3 \\ &Q(\ell,n) \leq 13 && \ell=1,\ldots,4 \; ; \; n=1,2,3 \\ &X(n) \leq 20 && n=1,2,3 \end{aligned} \qquad (6.20$$

In order to find non-inferior solutions, the trade-off functions, λ_{12} and λ_{13}, must be generated. The original problem is rewritten to use the ε-constraint approach. The decision maker is familiar with the order of the objective functions. The economic objective function is going to be used to minimize the objective functions; then the second and third functions will be changed to find the non-inferior solutions.

The Lagrangian L is formed, and the given data in the original equations will be replaced by known values.

$$L = -10\{\sum_{n=1}^{3}\sum_{\ell=1}^{4}[Q(\ell,n)+X(n)]/(1.08)^n\} + \sum_{n=1}^{3}\sum_{\ell=1}^{4}\frac{0.05}{1.08^n}[80+S(\ell,n)]Q(\ell,n)$$

$$+ 6\sum_{n=1}^{3} X(n)/(1.08)^n + \hat{\lambda}_{12}\{\sum_{n=1}^{3}[\sum_{\ell=1}^{4}\sum_{i=1}^{n}\psi(\ell,n-i+1)Q(\ell,i) + X(n)]$$

$$- \varepsilon_2\} + \hat{\lambda}_{13}\{\sum_{n=1}^{3}\sum_{\ell=1}^{4} P(n)[80+S(\ell,n)]Q(\ell,n) + 20\sum_{n=1}^{3} P(n)\, X(n)$$

$$- \varepsilon_3\} + \sum_{n=1}^{3}\mu_n\{R(n) - X(n) - \sum_{\ell=1}^{4} Q(\ell,n)\} +$$

$$\sum_{n=1}^{3}\mu_{n+3}\{\sum_{\ell=1}^{4}\sum_{i=1}^{n}\psi(\ell,n-i+1)Q(\ell,i) + X(n) - 35\} \tag{6.21}$$

The Lagrange multipliers, μ_1 to μ_6 belong to the constraints. The Kuhn Tucker necessary conditions for the minimum are:

$$\frac{\partial L}{\partial Q(\ell,n)} \geq 0 \qquad \forall \ell\ ,\ \forall n$$

$$Q(\ell,n)\frac{\partial L}{\partial Q(\ell,n)} = 0 \qquad \forall \ell\ ,\ \forall n$$

$$Q(\ell,n) \geq 0\ ,\ Q(\ell,n) \leq 13 \qquad \forall \ell\ ,\ \forall n \tag{6.22}$$

$$\frac{\partial L}{\partial \hat{\lambda}_{1j}} \leq 0 \qquad j=1,2$$

$$\hat{\lambda}_{1j} \geq 0 \qquad j=1,2$$

The problem with the above conditions was solved for several values of ε_2 and ε_3. The GRG computer program has been utilized successfully to solve the problem and to find the Lagrange multipliers λ_{12} and $\hat{\lambda}_{13}$.

The solution of the GRG computer program includes the Lagrange multipliers $\hat{\lambda}_{12}$ and $\hat{\lambda}_{13}$ for the constraints in the problem by using the ε-constraint approach.

To find the λ_{12} that belongs to the original problem the following is done:

$$\lambda_{12} = -\frac{\partial f_1}{\partial f_2} = -\frac{\partial \hat{f}_1}{\partial f_2} \cdot \frac{df_1}{d\hat{f}_1} \tag{6.23}$$

since $\hat{f}_1 = -f_1$

then $\lambda_{12} = -\frac{\partial \hat{f}_1}{\partial f_2} \cdot (-1) = -\hat{\lambda}_{12} = \frac{\partial \hat{f}_1}{\partial f_2}$

This is acceptable because when the amount of f_2 increases, then the marginal benefits will also increase; as a result, λ_{12} will be negative. All this means that if a unit of f_2 is decreased, λ_{12} units of economical benefits are decreased.

A similar process is utilized to find λ_{13}.

$$\lambda_{13} = -\frac{\partial f_1}{\partial f_3} = -\frac{\partial \hat{f}_1}{\partial f_3} \cdot \frac{df_1}{d\hat{f}_1} \tag{6.24}$$

then

$$\lambda_{13} = -\frac{\partial \hat{f}_1}{\partial f_3} \cdot (-1) = -\hat{\lambda}_{13} = \frac{\partial \hat{f}_1}{\partial f_3}$$

The non-inferior solution including these steps is shown in the first five columns of Table 6.2.

The unit of f_2 is an acre ft/day, and the units of f_3 are ft. acre ft/day. The values chosen were large enough so that the decision-maker would have a good idea of the variation of the Lagrange multipliers and levels of the objective functions.

The values of the surrogate worth function, W_{12} and W_{13}, are shown in Table 6.2. They were generated by the decision-maker. To do so, he was asked whether he would give up one unit of (f_2) to get a marginal increment of λ_{12} units of benefit-cost, considering the different levels of the objective functions. When he said yes, he was asked to assign a numerical value between 0 and +10 to show how strongly he felt about that trade (zero being indifferent). When he said no, he was asked to assign a value between 0 and -10. Then the surrogate worth function W_{12} was developed . To develop W_{13}, the same process was repeated. Again, the decision-maker was asked whether he would give up one unit of energy consumption to get a marginal increment of λ_{13} units of benefit-cost, considering the different levels of the objective functions.

In Table 6.2, one set of trade-offs is in the indifference band, $W_{12} = 0$ and $W_{13} = 0$. Solution number 15 is a non-inferior one that belongs to the indifference band. Since one of the non-inferior points had both worth functions equal to zero, possible multiple regressions to approximate $W_{12}(\varepsilon_2, \varepsilon_3)$ and $W_{13}(\varepsilon_2, \varepsilon_3)$ were not done.

Table 6.2

NONINFERIOR POINTS AND DECISION-MAKER RESPONSES

Solution No	f_1	ε_2	ε_3	λ_{12}	λ_{13}	W_{12}	W_{13}
1	605.63	70.0	1700.0	-2.96	-0.068	10	-10
2	564.44	60.0	1700.0	-3.26	-0.061	10	-10
3	539.05	50.0	1700.0	-3.56	-0.058	5	-10
4	509.24	40.0	1700.0	-3.59	-0.048	0	-10
5	488.83	30.0	1700.0	-3.63	-0.031	-2	-8
6	435.75	20.0	1700.0	-5.17	-0.009	-7	-8
7	585.95	70.0	1500.0	-2.77	-0.126	10	-10
8	557.90	60.0	1500.0	-2.96	-0.113	10	-10
9	526.66	50.0	1500.0	-3.13	-0.104	5	-10
10	494.84	40.0	1500.0	-3.47	-0.103	0	-10
11	460.17	30.0	1500.0	-3.48	-0.091	-2	-8
12	424.56	20.0	1500.0	-3.72	-0.092	-7	-8
13	406.51	16.0	1500.0	-5.59	-0.079	-10	-8
14	492.69	70.0	1000.0	-0.47	-0.223	2	0
15	471.41	60.0	1000.0	-2.84	-0.219	0	0
16	440.96	50.0	1000.0	-2.98	-0.214	-2	0
17	407.56	40.0	1000.0	-3.12	-0.240	-7	0
18	376.79	31.25	1000.0	-5.31	-0.347	-10	0
19	372.39	65.0	600.0	-1.18	-0.399	-10	5
20	365.02	62.0	600.0	-2.78	-0.383	-10	5

Table 6.3

NON-INFERIOR SOLUTIONS

Solution No.	QUANTITY OF PUMPING WATER FROM L-TH CELL AT THE N-TH PERIOD L=2 N 1	2	3	L=4 N 1	2	3	L=5 N 1	2	3	L=6 N 1	2	3
1	3.439	2.360	3.206	13.0	13.0	13.0	13.0	6.981	13.0	13.0	12.44	0.0
2	3.658	2.544	3.404	13.0	13.0	13.0	13.0	6.993	13.0	13.0	12.60	0.0
3	3.912	2.761	4.161	13.0	13.0	13.0	13.0	7.085	13.0	13.0	13.0	0.0
4	4.114	2.968	4.503	13.0	13.0	13.0	13.0	8.038	13.0	13.0	12.93	0.0
5	4.805	3.437	5.117	13.0	13.0	13.0	13.0	10.173	13.0	13.0	13.00	0.0
6	5.027	3.611	6.000	13.0	13.0	13.0	13.0	6.050	13.0	13.0	13.00	0.0
7	2.657	1.573	1.777	13.0	13.0	13.0	13.0	8.235	13.0	13.0	11.40	0.0
8	2.908	1.795	2.203	13.0	13.0	13.0	13.0	5.328	13.0	11.62	7.88	0.0
9	3.099	1.944	2.549	13.0	13.0	13.0	13.0	5.103	13.0	11.954	8.63	0.0
10	3.002	2.182	2.924	13.0	13.0	13.0	13.0	6.006	13.0	10.894	9.81	0.0
11	3.048	2.209	3.212	13.0	13.0	13.0	13.0	6.071	13.0	11.93	9.71	0.0
12	3.211	2.094	3.465	13.0	13.0	13.0	13.0	3.514	13.0	12.43	12.39	0.0
13	3.338	2.761	5.853	13.0	13.0	13.0	13.0	3.842	13.0	8.47	11.39	0.0
14	1.314	0.761	0.687	13.0	9.399	7.497	11.12	2.63	6.54	6.78	4.41	0.0
15	1.336	0.525	0.856	13.0	9.564	9.490	10.97	0.00	10.29	6.28	3.35	0.0
16	1.469	0.810	0.931	13.0	9.232	10.341	10.67	0.00	10.34	5.78	3.39	0.0
17	1.098	0.0	0.548	13.0	8.769	12.523	13.0	0.0	10.66	4.93	2.37	0.0
18	2.272	1.193	1.030	13.0	11.96	11.523	13.0	0.0	12.13	1.72	0.0	0.0
19	0.0	0.407	0.519	5.61	5.64	5.840	4.138	2.274	5.64	1.97	2.67	0.0
20	0.424	0.413	0.543	6.42	7.18	5.663	4.101	0.0	5.79	0.779	3.40	0.0

Table 6.4

NON-INFERIOR SOLUTIONS

WATER TAKEN FROM THE STREAM AT THE N-TH PERIOD			
Sol. No.	N 1	2	3
1	20.0	20.0	10.06
2	0.0	19.92	20.00
3	7.79	20.0	1.838
4	17.5	0.0	1.49
5	6.56	0.0	0.932
6	0.0	0.0	0.0
7	20.0	20.0	14.355
8	20.0	20.0	3.79
9	20.0	9.97	3.45
10	19.64	0.0	3.075
11	9.51	0.0	2.787
12	0.0	0.0	2.534
13	0.0	0.0	0.146
14	20.0	20.0	20.0
15	20.0	20.0	12.36
16	14.59	17.58	10.38
17	4.99	19.85	8.26
18	0.0	18.92	7.30
19	20.0	20.0	20.0
20	18.36	20.0	20.0

The decision variables corresponding to the optimal preferred solutions and the objective functions are:

n	$Q^*(\ell,n)$ ℓ = 2	4	5	6	X(n)
1	1.336	13.0	10.97	6.28	20.0
2	0.525	9.564	0.0	3.35	20.0
3	0.856	9.49	9.29	0.0	12.36

$f_1^* = 471.41$ \$/day

$f_2^* = 60.0$ acre ft/day

$f_3^* = 1000.0$ ft. acre ft/day

6.5 ANALYSIS OF RESULTS AND OBSERVATIONS

The first results obtained by following the SWT method were the minimum and maximum levels of the second and third objective functions. The minimum level of water taken from the stream is obtained when the pumping water satisfied the minimum demand, and no stream water supply was imposed. The minimum level of energy consumption is met at the point where the water to be supplied from the stream reaches its maximum capacity.

The following analysis is required to study the behavior of the objective functions with respect to each other. Using the results shown in Table 8, it is observed that for large quantities of water taken from the stream that the stream water supply tends to satisfy its maximum capacity, and the pumping water supply tends to decrease.

In addition, the opposite also holds. As the quantity of water taken from the stream decreases, the quantity of pumping water increases, and the stream water decreases.

The influence of the energy consumption quantity over the decision variables is as follows. The quantity of pumping water will tend to increase as energy consumption increases. The behavior of the stream water supply with respect to energy consumption is such that the total stream water increases as this objective decreases.

Graph #5 is created with the data from Table 6.5. This graph shows that the total stream water supply and the total extra water produced increases as the water taken from the stream increases, for a constant energy consumption. The opposite happens with the total infiltration and the total pumping water.

The behavior of the economic objective function with respect to the environmental objective function, is such that f_1 decreases approximately linearly as f_2 decreases, (see Graph #1). The economic objective function also decreases as the energy consumption objective decreases, maintaining constant f_2. (see Graph #2).

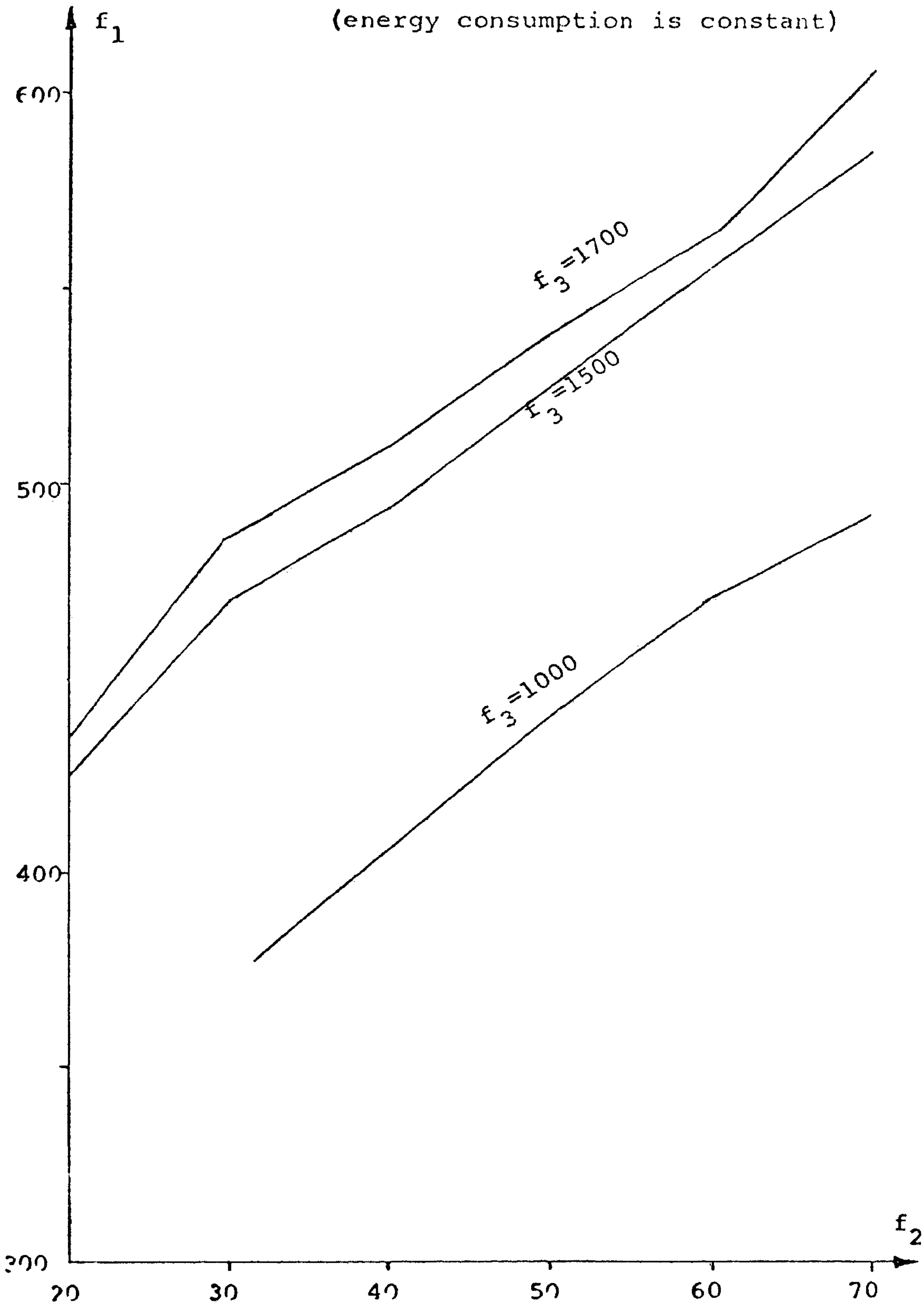
The Lagrange multiplier λ_{12} or the trade-off between the first and second objective functions as a function of f_2 shows that λ_{12} decreases as the water taken from the stream decreases. The decreasing slope tends to be higher as the second objective is close to its minimum level, (see Graph #3). The Lagrange multiplier λ_{13} as a function of the third objective function is shown in Graph #4. Here, the slope tends to be a constant value. The λ_{13} increases as f_3 increases.

As part of the Surrogate Worth Trade-off method all this information was provided to the decision-maker. He was a graduate student who works in the group of water resources system. Because of the form of the objective functions and the data provided, the results were expected by the decision-maker.

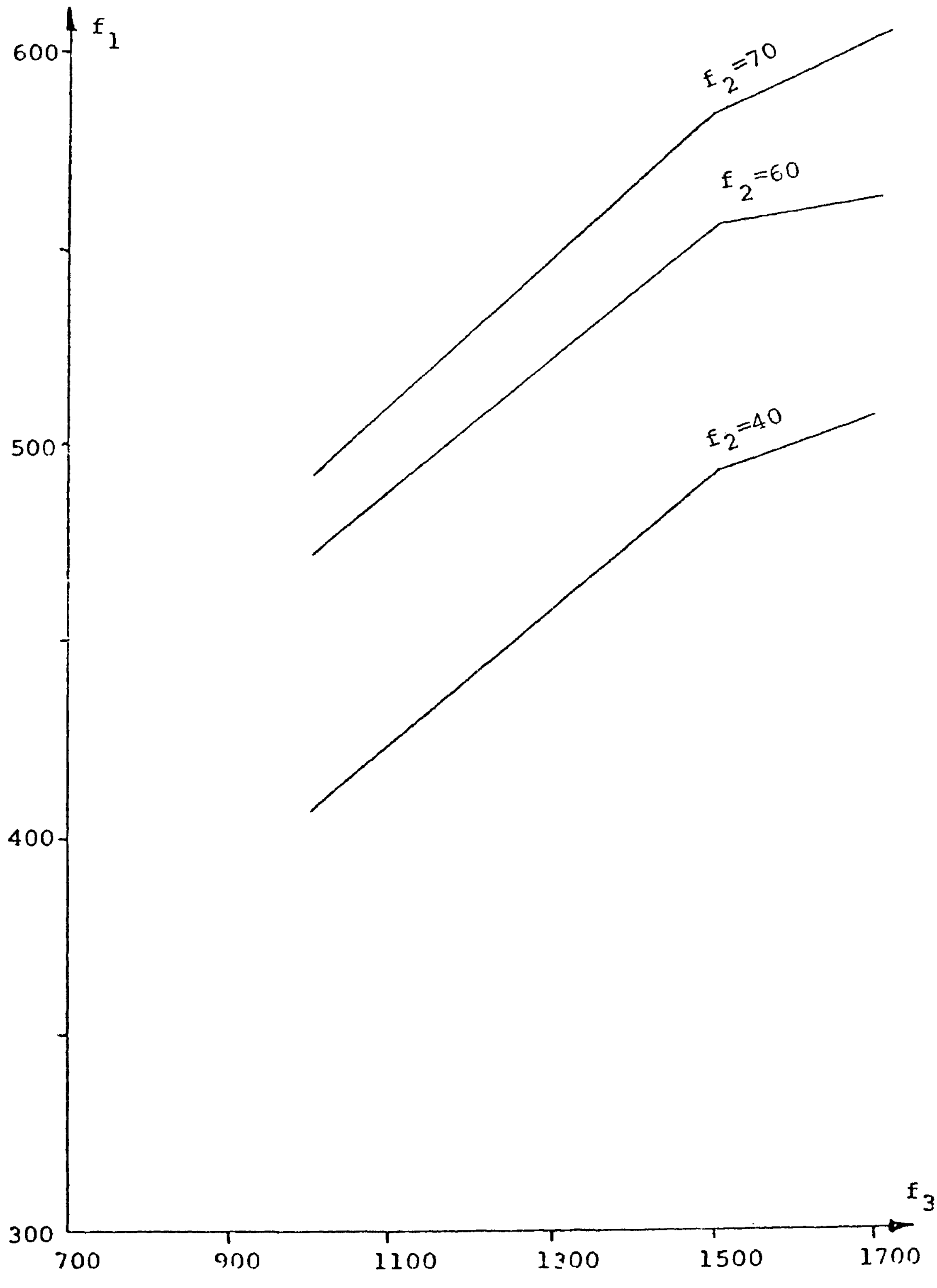
This three non-commensurable objective problem has been solved satisfactorily by using the SWT method. Twenty non-inferior points were necessary to obtain to create the information explained above. Each of these points took an average of 8 seconds of computer time in the UNIVAC 1108. In summary, the total real time was approximately 4 minutes. This includes the calculations to find the non-inferior points and some inferior points as part of the ε-constraint approach.

The GRG computer program was used to find the non-inferior points. This program required a subroutine called GCOMP which includes the objective functions and constraints.

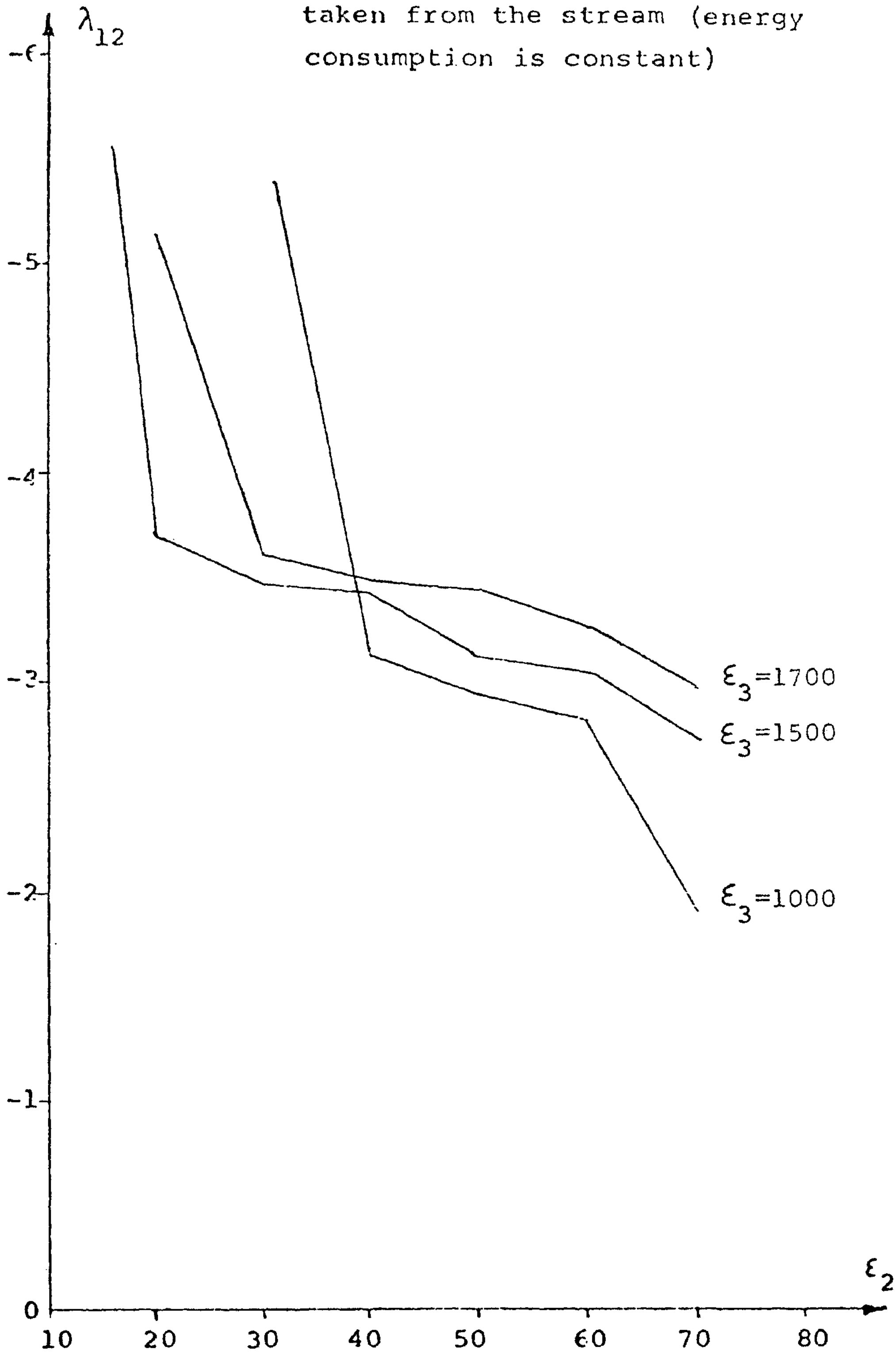
Graph #1: Benefit-cost function vs. quantity of water taken from the stream (energy consumption is constant)



Graph #2: Benefit-cost function vs. energy consumption (water taken from the stream is constant)



Graph #3: The Lagrange multiplier λ_{12} as a function of the quantity of water taken from the stream (energy consumption is constant)



Graph #4: The Lagrange multiplier λ_{13} as a function of the energy consumption (water taken from the stream is constant)

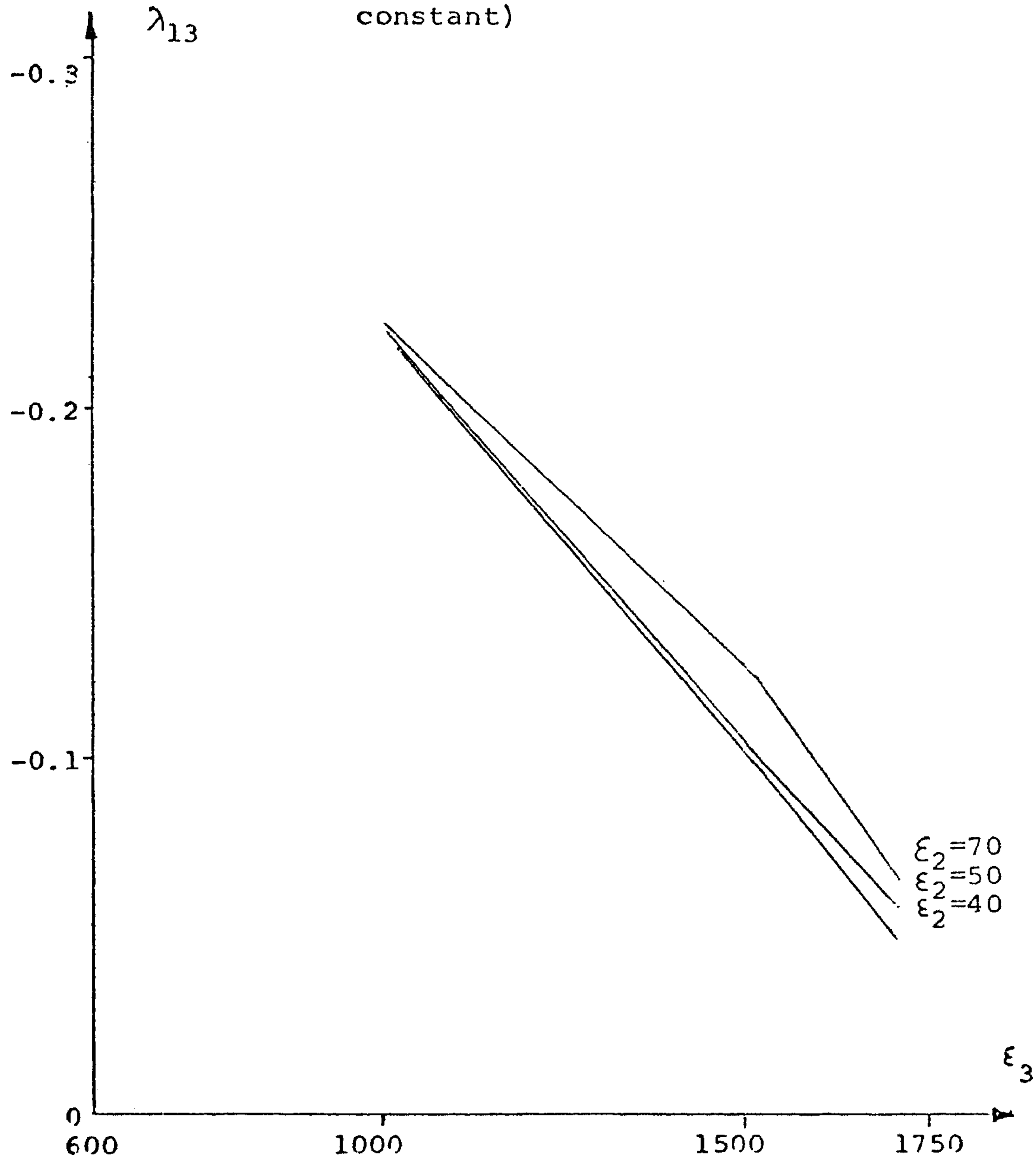
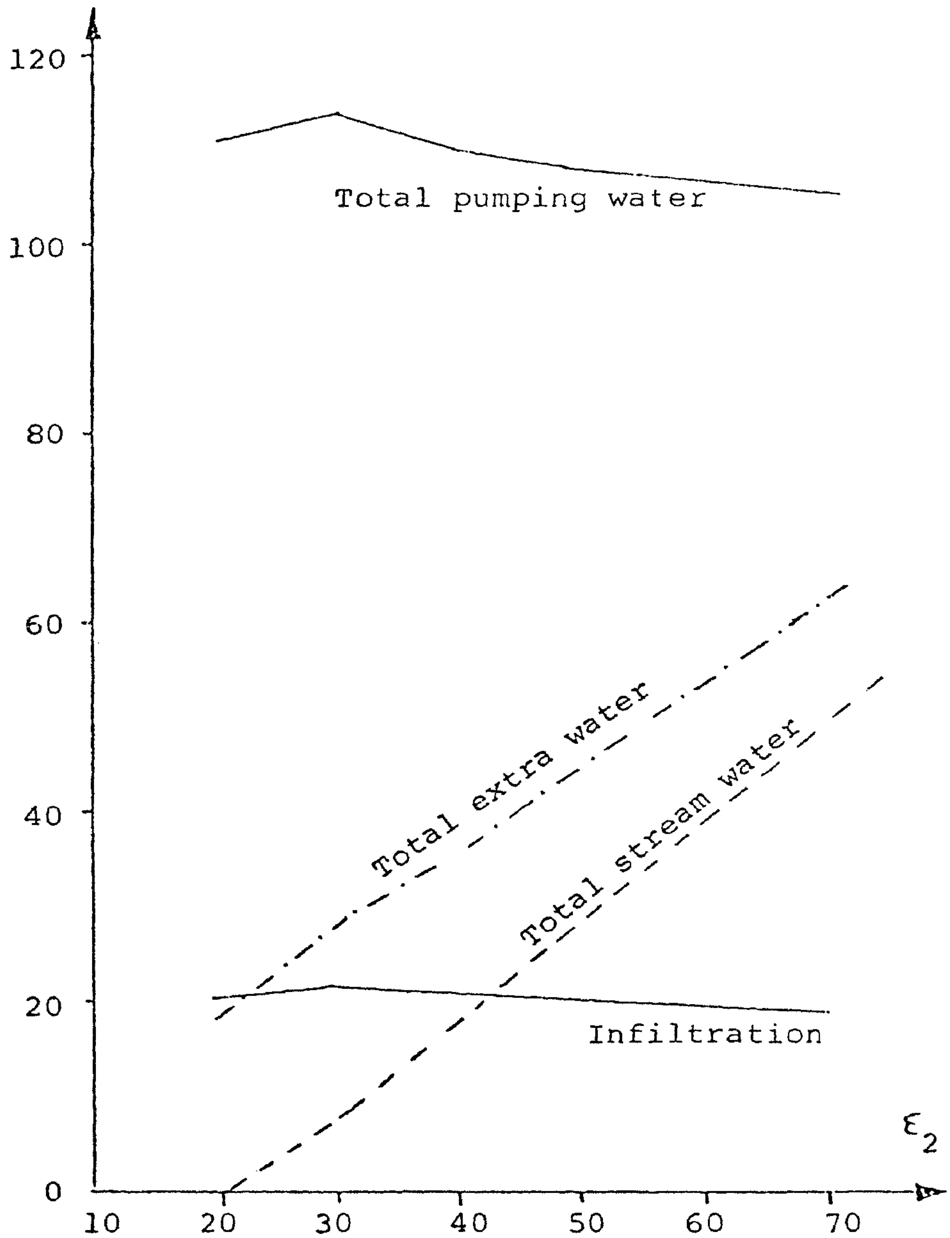


Table 6.5

DATA FOR WATER SUPPLY AND PUMPAGE

The figures are given in acre ft/day

Sol. No.	Total Stream Water Supply	Total Pumping Water	Total Extra Water	Infiltration
1	50.06	106.44	63.50	19.94
2	39.92	107.21	54.13	20.08
3	29.63	108.92	45.55	20.37
4	18.99	110.57	36.56	21.01
5	7.49	114.49	28.98	22.51
6	0.0	111.69	18.69	20.00
7	54.36	94.88	56.24	15.74
8	43.79	96.76	47.55	16.21
9	33.42	98.30	38.72	16.58
10	22.72	99.82	29.54	17.28
11	12.30	101.20	20.50	17.70
12	2.53	102.11	11.64	17.47
13	0.15	100.66	7.81	15.85
14	60.00	64.16	31.16	10.00
15	52.36	64.69	24.05	7.64
16	42.55	65.99	15.54	7.45
17	33.10	66.92	7.02	6.90
18	26.22	66.78	0.0	5.03
19	60.00	34.72	1.72	5.00
20	58.36	34.73	0.09	3.64



Graph # 5: Totals of stream water supply, pumping water, extra water, and infiltration (ε_3=1700)

REFERENCES

1. Bates, H., "Computer Code for Wolfe Algorithm," Kansas State University, 1970.

2. Bear, J., Dynamics of Fluids in Porous Media. American Elsevier Publishing Company, N.Y., 1972.

3. Bear, J., U. Shamir and E.A. Hefez, "Numerical Modelling of Groundwater Systems." P.N. 180 Technion, Haifa, Israel, June 1972.

4. Bredehoeft, J.D. and G.F. Pinder, "Digital Analysis of Areal Flow in Multiaquifer Groundwater Systems: A Quasi Three-Dimensional Model," Water Resources Research, 6(3), 883-888, 1970.

5. Bryson, A.E., Jr., and Y.C. Ho, Applied Optimal Control, Ginn and Company, Waltham, Mass., 1969.

6. Buras, N., "Conjunctive Operation of Dams and Aquifers." ASCE-J. Hydraulics Division, 89(6): 111-131, 1963.

7. Cohon, J.L., and D.H. Marks, "A Review and Evaluation of Multi-objective Programming Techniques," Water Resources Research, 11(2), 208-220, 1975.

8. Dreizin, Y. C., "Applications of the Superposition Approach to the Modeling and Management of Ground and Surface Water Systems," Ph.D. Dissertation, Case Western Reserve University, 1971.

9. Falkenbarg, D.R., "The Identification of Distributed Parameter Systems," Ph.D. Dissertation, Case Western Reserve University, 1971.

10. Hadley, G., Nonlinear and Dynamic Programming, Reading, Mass., Addison-Wesley, 1964.

11. Haimes, Y.Y., "Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water, Phase I," Office of Water Res. Research, U.S. Dept. of Interior, Wash., D.C., PB 226-869, 1973.

12. Haimes, Y.Y., "Integrated System Identification and Optimization for Conjunctive Use of Ground and Surface Water, Phase II," Office of Water Resources Research, U.S. Dept. of Interior, Wash., D.C., NTIS-PB 238-891, 1974.

13. Haimes, Y.Y., W.A. Hall and H.T. Freedman, Multiobjective Optimization in Water Resources Systems: The Surrogate Worth Trade-off Method, Elsevier Scientific Publishing Company, The Netherlands, 1975.

14. Haimes, Y.Y. and W.A. Hall, "Multiobjectives in Water Resources Systems Analysis: The Surrogate Worth Trade-off Method," Water Resources Research, vol. 10, no. 4, pp. 615-624, August 1974.

15. Haimes, Y.Y. and D. Macko, "Hierarchical Structures in Water Resources Systems Management." IEEE-SMC, SMC-3(4), 396-402, July 1973.

16. Haimes, Y.Y., R.L. Perrine, and D.A. Wismer, "Identification of Aquifer Parameters by Decomposition and Multilevel Optimization." Israel Journal of Technology, 6(5), 322-329, 1968.

17. Howe, C.W., Benefit-Cost Analysis for Water Systems Planning, American Geophysical Union, Washington, D.C., 1971.

18. Jacob, C.E.,"Flow of Ground Water," Chapter V in Engineering Hydraulics, Ed., H. Rouse, John Wiley and Sons, Inc., New York, pp. 321-386, 1950.

19. Kaplan, M., "The Planning and Operation of a Regional Water Quality Management System: A Multilevel Approach," M.S. Thesis, Systems Engineering Dept., Case Western Reserve University, 1972.

20. Karplus and Kawameto, "Identification Problem in Distributed Parameter Systems," Short Course Notes at UCLA, January 1966.

21. Kleinecke, D.R., "Use of L.P. for Estimating Geohydrological Parameters of Groundwater Basins," Water Resources Research, vol. 7, no. 2, 1971.

22. Kuester, J.L., and J.H. Mize, Optimization Techniques with Fortran, McGraw-Hill, New York, 1973.

23. Lasdon, L., Optimization Theory for Large Systems, Macmillan Co., London, 1970.

24. Lions, J.L., Optimal Control of Systems Governed by Partial Differential Equations, Springer-Verlag, Berlin, 1971.

25. Lopez, H., "Modeling and Identification of Groundwater Systems," M.S. Thesis, Case Western Reserve University, Cleveland, Ohio, 1973.

26. Lopez, H., Y.Y. Haimes and P. Das, "Nonlinear Estimation of Distributed Parameter of Groundwater Systems," To appear in Journal of Hydrology, 1975.

27. Maddock, T. III, "A Program to Simulate an Aquifer Using Alternating Direction Implicit-Iterative Procedure," U.S. Dept. of Interior, Geological Survey, 1969.

28. Maddock, T. III, "Management Model as a Tool for Studying the Worth of Data," Water Resources Research, 9(2), 270-280, 1973.

29. Maddock, T. III, "The Operation of a Stream-Aquifer System under Stochastic Demands," Water Resources Research, 10(1), February, 1974.

30. Maddock, T. III, and Y.Y. Haimes, "A Tax-Quota System for the Planning and Management of Groundwater," Water Resources Research, vol. 11, no. 1, p. 7, February 1975.

31. Marino, M.A. and Yeh, W. W-G., "Identification of Parameters in Finite Leaky Aquifer System," Journal of Hydraulics Division, ASCE, 99(2), pp. 319-336, February 1973.

32. Marquardt, D.W., "An Algorithm for Least-Square Estimation of Non-Linear Parameters," J. Soc. Indus. Appl. Math., Vol. II, no. 2, June 1963.

33. Mikhlin, S.G., Mathematical Physics, An Advanced Course. North Holland Publishing Co., pp. 418-420, 1970.

34. Peaceman, D.W. and H.H. Rachford, Jr., "The Numerical Solution of Parabolic and Elliptical Difference Equations," J. Soc. Indust. Appl. Math., 3(11), 28-41, 1955.

35. Pinder, G.F. and J.D. Bredehoeft, "Application of the Digital Computer for Aquifer Evaluation," Water Resources Research, 4(5), 1069-1109, 1968.

36. Phillipson, G.A., "Identification of Distributed Systems," American Elsevier, New York, 1971.

37. Plummer, P.M., "Observation and Evaluation of Water Resources in the Hamilton to New Baltimore Area, 1971-1973," The Miami Conservancy District, Dayton, Ohio, August 1974.

38. Prickett, T.A. and C.G. Lonquist, "Selected Digital Computer Techniques for Groundwater Resource Evaluation," Illinois State Water Survey Division, Urbana, Illinois, Bulletin 55, 1971.

39. Roach, G.F., Green's Functions, Introductory Theory with Applications, Van Norstad Reinhold Co., London, 1970.

40. Spieker, A.M., "Groundwater in the Lower Great Miami River Valley, Ohio," U.S. Government Printing Office, No. 605-A,B,C,D., Wash. D.C., 1968.

41. Tyson, H.N. and E.M. Weber, "Groundwater Management for the Nation's Future. Computer Simulation at Groundwater Basins," ASCE, Hydraulics Division, vol. 90, pp. 59-78, 1964.

42. Wolfe, P., "The Simplex Method for Quadratic Programming," Econometrica, 27, 382-398, 1959.

43. Yeh, W.G. and G.W. Tauxe, "Quasilinearization and Identification of Aquifer Parameters," Water Resource Research, vol. 7, no. 2, April 1971.

44. Yu, W. and Y.Y. Haimes, "Multilevel Optimization for Conjunctive Use of Ground and Surface Water," Water Resources Research, 10(4), 625-636, August 1974.